Musterlösung: Aufgabe 11

a) $N = 2 \sum_{k<k_F} 1 = \frac{2V}{(2\pi)^d} \int_{k<k_F} d\vec{k} = \frac{2V}{(2\pi)^d} \cdot \frac{\pi^{d/2} k_F^d}{\Gamma(d/2+1)}$

Das Volumen der n-dimensionalen Kugel vom Radius R

$$V_n = \frac{\pi^{n/2} R^n}{\Gamma(n/2+1)}$$

Oberfläche: $S_n = \frac{n V_n}{R} = \frac{n \pi^{n/2} R^{n-1}}{\Gamma(n/2+1)}$

$$\Rightarrow n = N/V = \frac{2}{\Gamma(d/2+1)} \left(\frac{k_F^2}{4\pi}\right)^{d/2}$$

$$\Rightarrow k_F = 2\pi^{1/2} \left(\frac{1}{2} n \Gamma(d/2+1)\right)^{1/d}$$

$$\varepsilon_F = \frac{\hbar^2}{2m} k_F^2 = \frac{2\pi\hbar^2}{m} \left(\frac{1}{2} n \Gamma\left(\frac{d}{2}+1\right)\right)^{2/d} = A_d\, n^{2/d}$$

$$A_d = \frac{2\pi\hbar^2}{m} \left(\frac{\Gamma(d/2+1)}{2}\right)^{2/d}$$

b) $\nu(\varepsilon) = \frac{1}{V} \sum_{\vec{k}} \delta\left(\varepsilon - \frac{\hbar^2 k^2}{2m}\right) = \frac{1}{(2\pi)^d} \int d\vec{k}\, \delta\left(\varepsilon - \frac{\hbar^2 k^2}{2m}\right) =$

1. Bz

$$= \frac{1}{(2\pi)^d} \frac{m}{\hbar^2} \left(\frac{\hbar^2}{2mE}\right)^{1/2} \cdot \frac{d\, \pi^{d/2}}{\Gamma(d/2+1)} \left(\frac{2mE}{\hbar^2}\right)^{\frac{d-1}{2}} =$$

$$= \frac{d}{2\Gamma(d/2+1)} \left(\frac{m}{2\hbar^2\pi}\right)^{d/2} \varepsilon_F^{d/2-1} \left(\frac{\varepsilon}{\varepsilon_F}\right)^{d/2-1} =$$

$$= \frac{dn}{4\varepsilon_F} \left(\frac{\varepsilon}{\varepsilon_F}\right)^{d/2-1}$$

$$\Rightarrow \nu(\varepsilon_F) = \frac{dn}{4\varepsilon_F}$$