Computer simulations in statistical physics
MC for 2D Ising model using Wolff updates

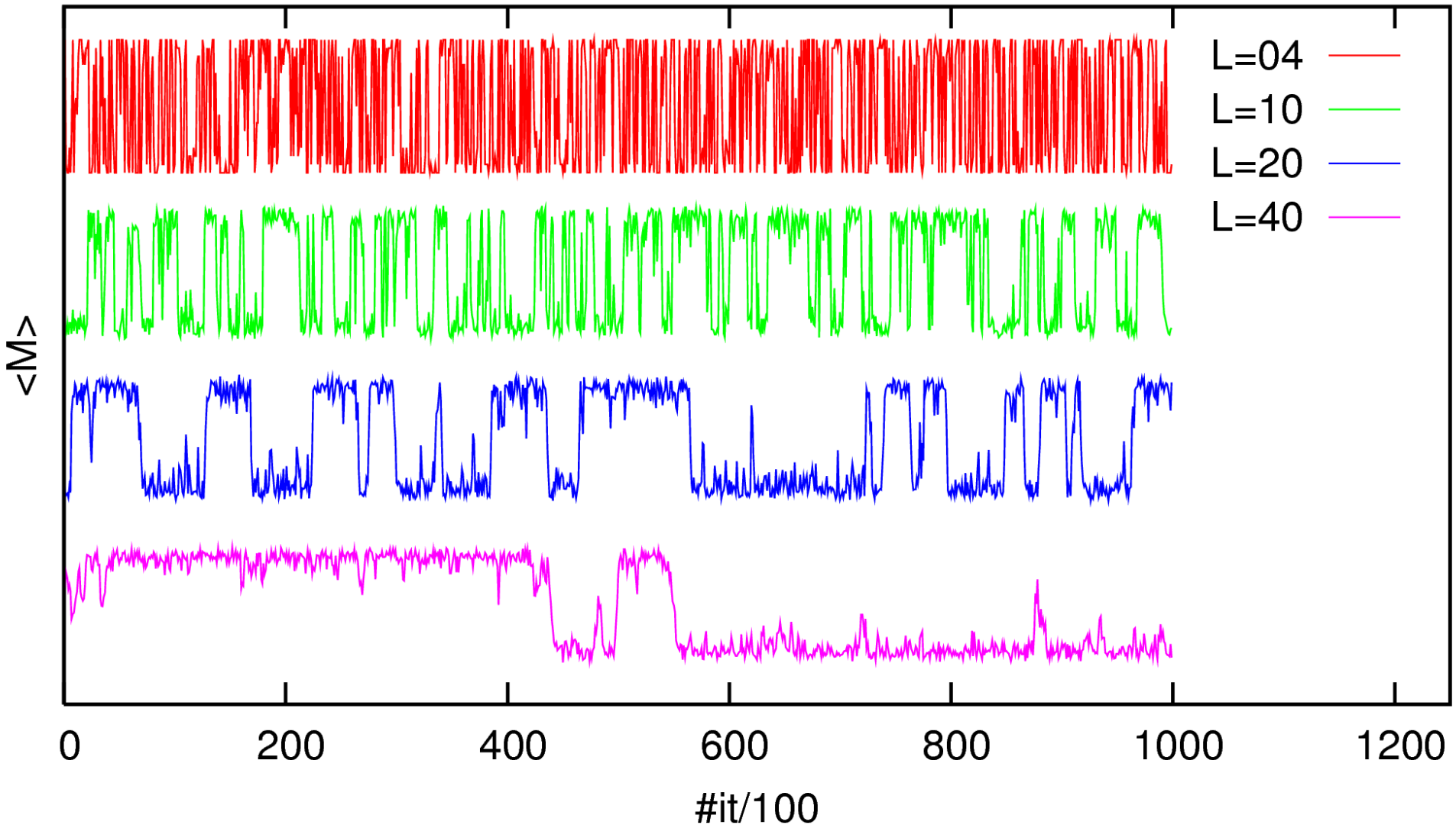
Raw data: traces, covariance and autocorrelation time

Simple observables: $|M|$, M^2, E, E^2

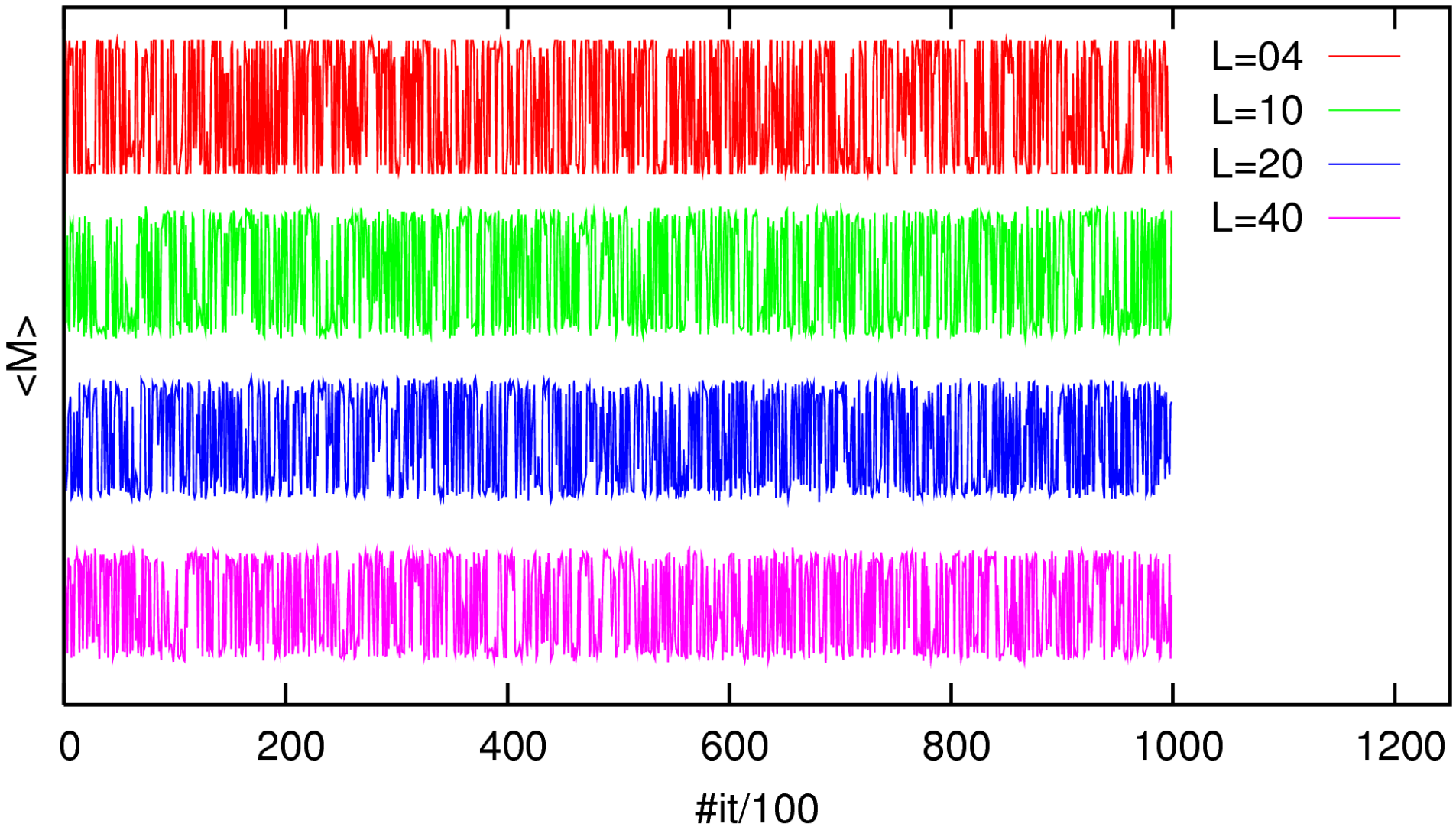
Smart observables: susceptibility, specific heat & Binder's cumulant

Raw data: traces, covariance and autocorrelation time

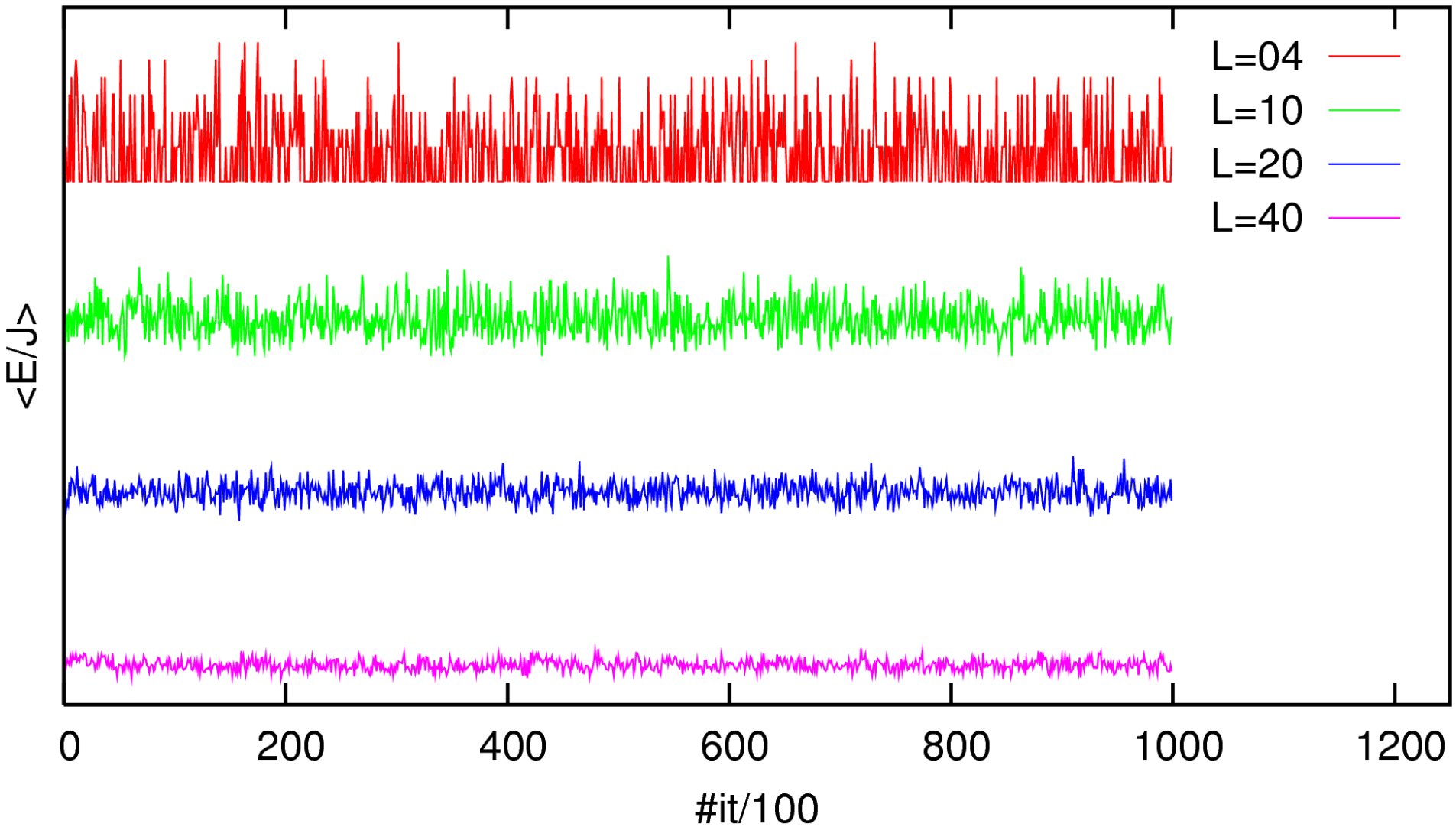
Trace: magnetization for $T = 2.27 J/k_B \approx T_C$ (10^5 sweeps)



Trace: magnetization for $T = 2.27 J/k_B \approx T_C$ (10^5 sweeps)- Wolff updates

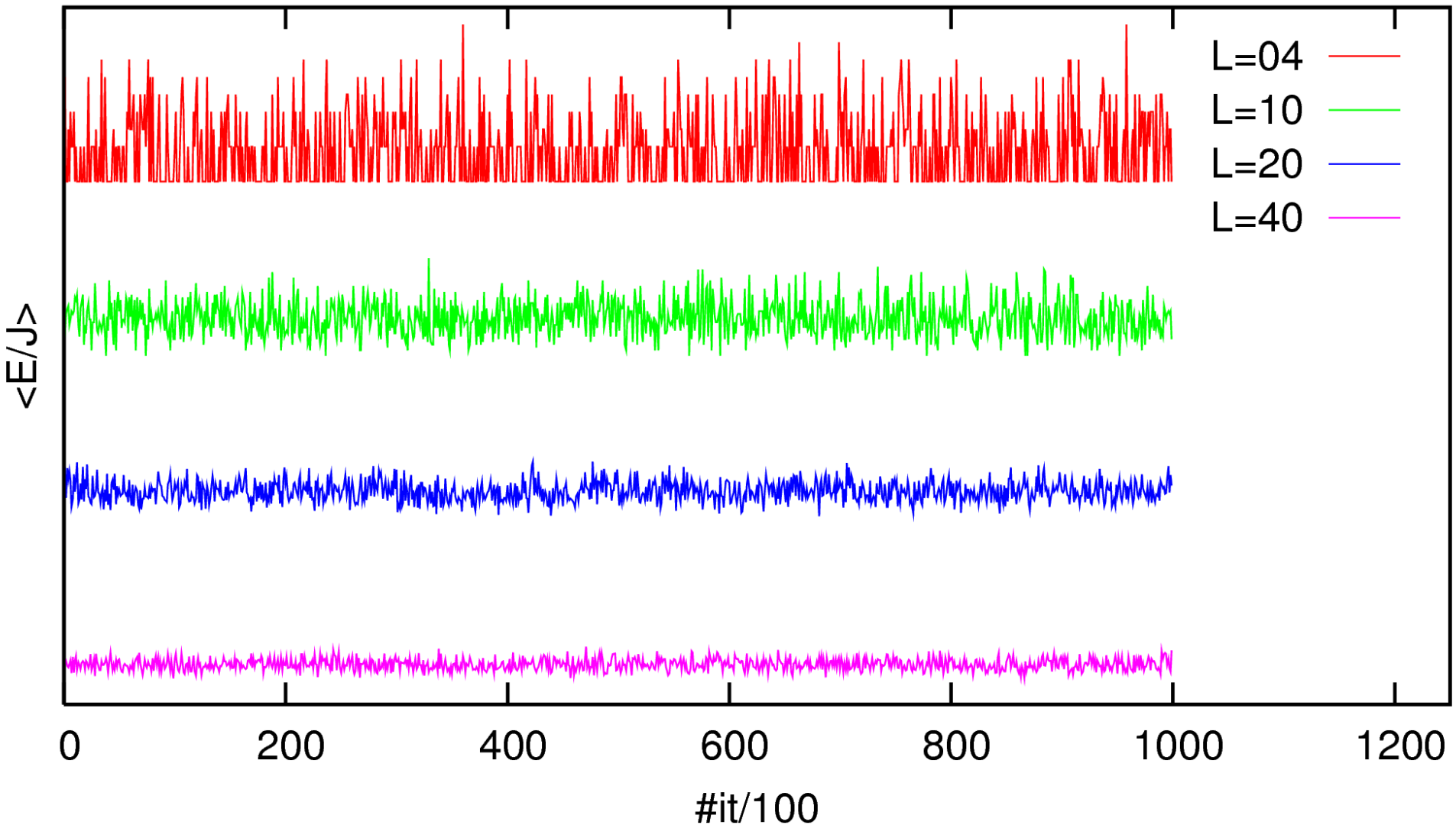


Trace: energy for $T = 2.27 J/k_B \approx T_C$ (10^5 sweeps)



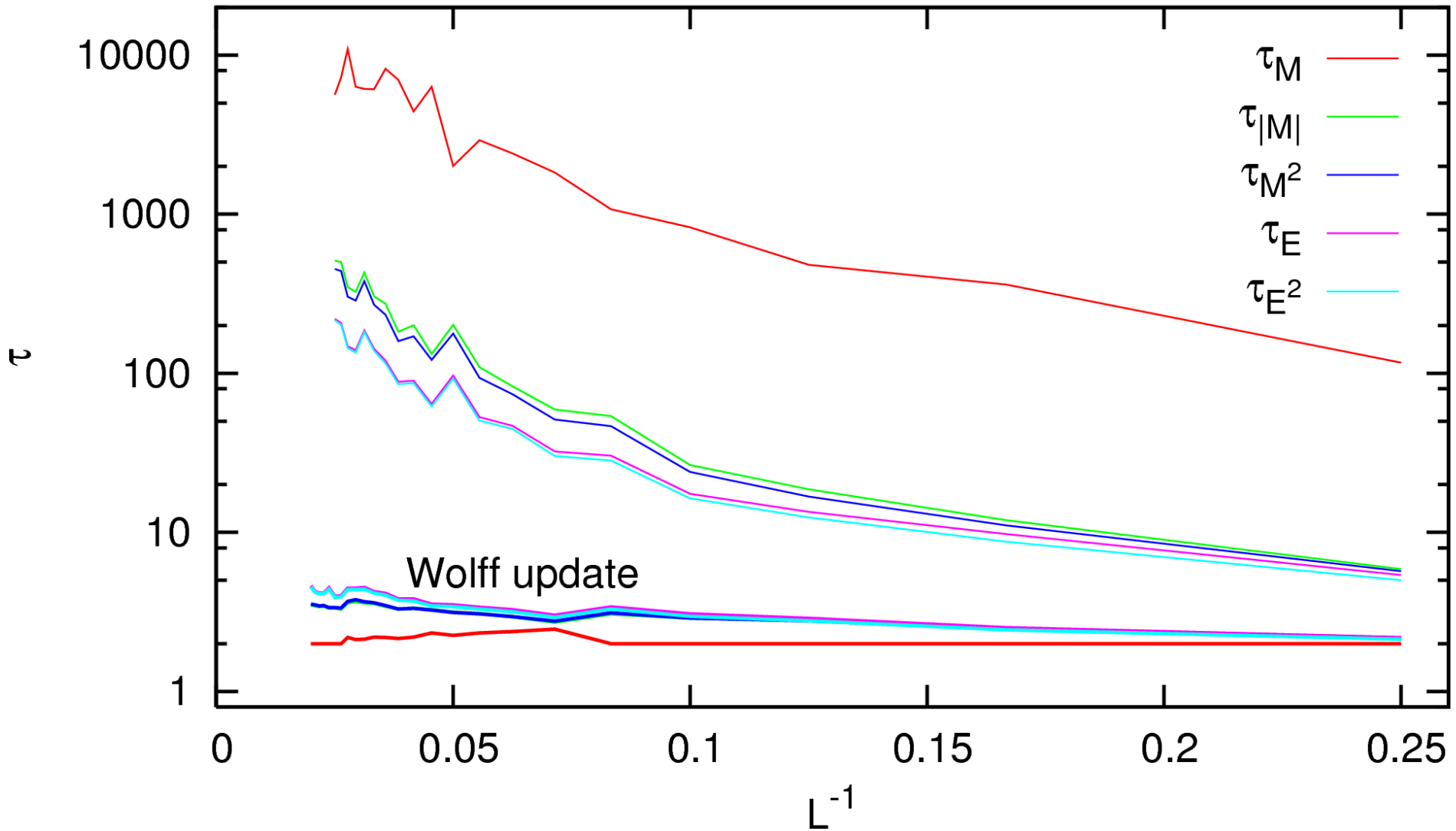
Very different shapes for traces of M and E; here less variation for larger L.

Trace: energy for $T = 2.27 J/k_B \approx T_C$ (10^5 sweeps) - Wolff updates



Low energy $\rightsquigarrow$ large M, but flat region for $|< M >| \lesssim 0.6$

Autocorrelation times for $T = 2.27 J/k_B \approx T_C$ (10^5 sweeps)



Autocorrelation times important, specific to observable, increase with L.

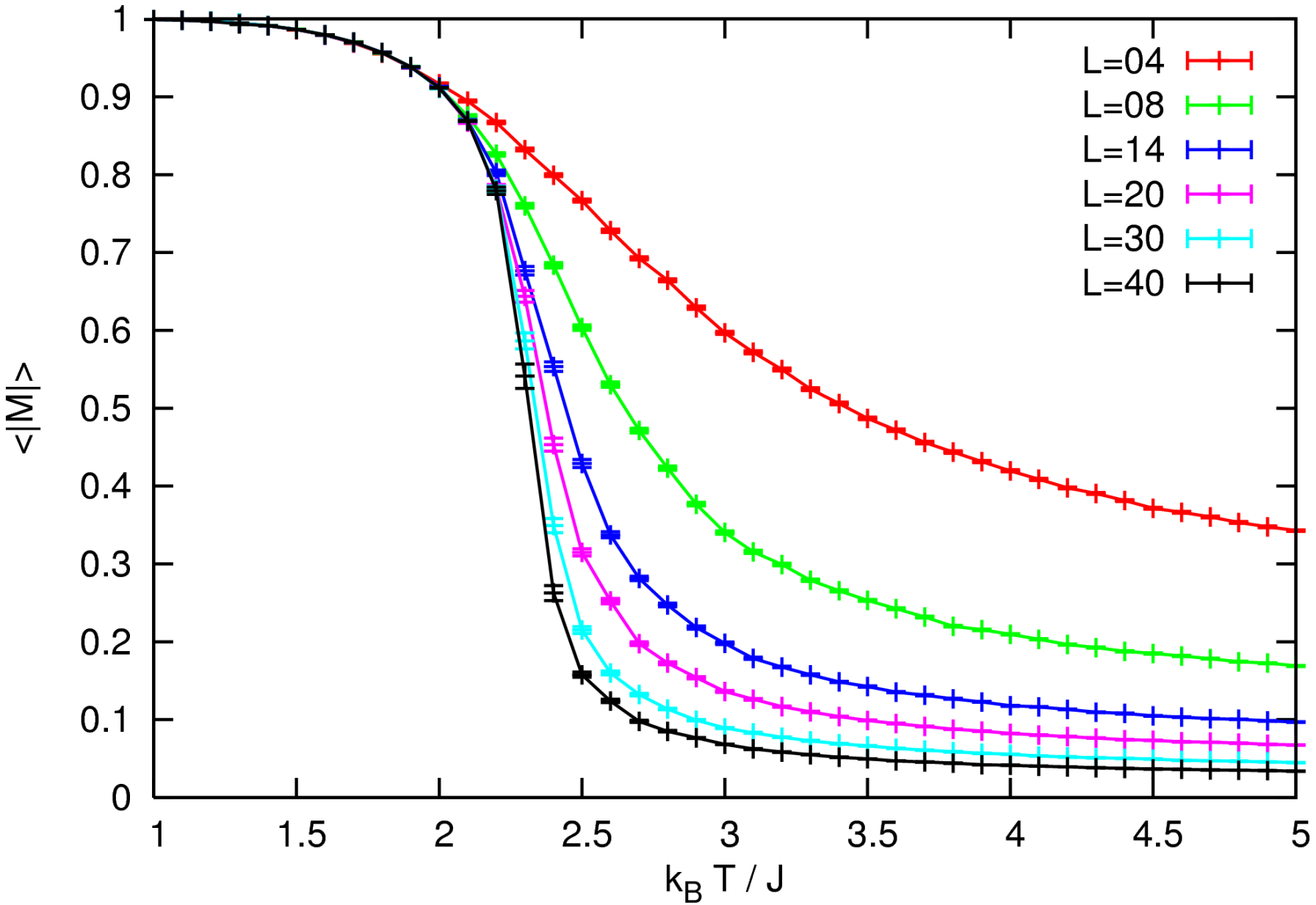
Simple observables: $|M|$, M^2, E, E^2

The following results have been obtained from Monte Carlo simulations of the 2d square lattice Ising model using single-spin-flip Metropolis updates.

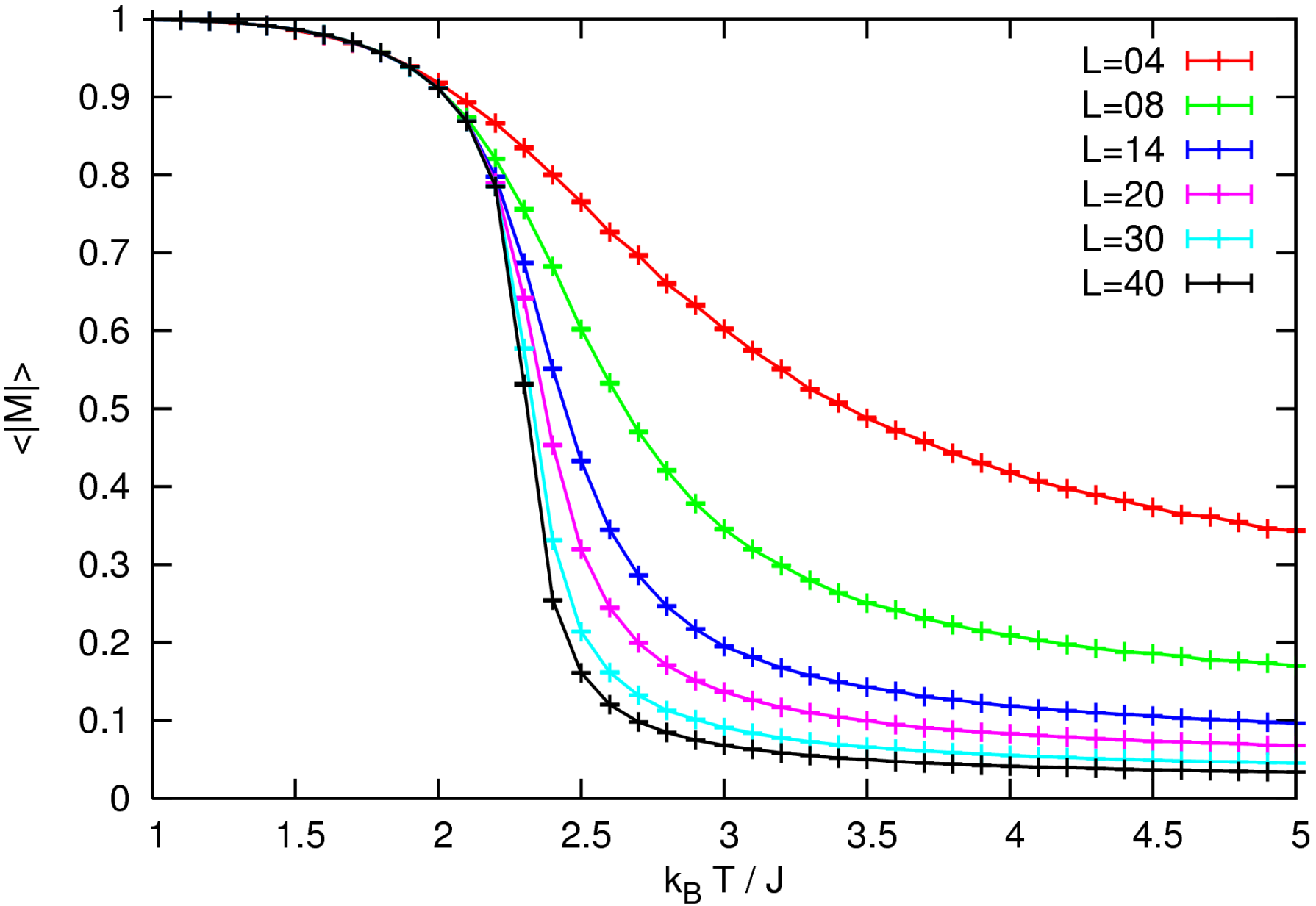
Let us first look at the obvious observables: magnetization and energy (plus their squares).

All observables are scaled per number of lattice sites, e.g. $\langle E \rangle \equiv \langle H \rangle / N; \quad N = L^2$

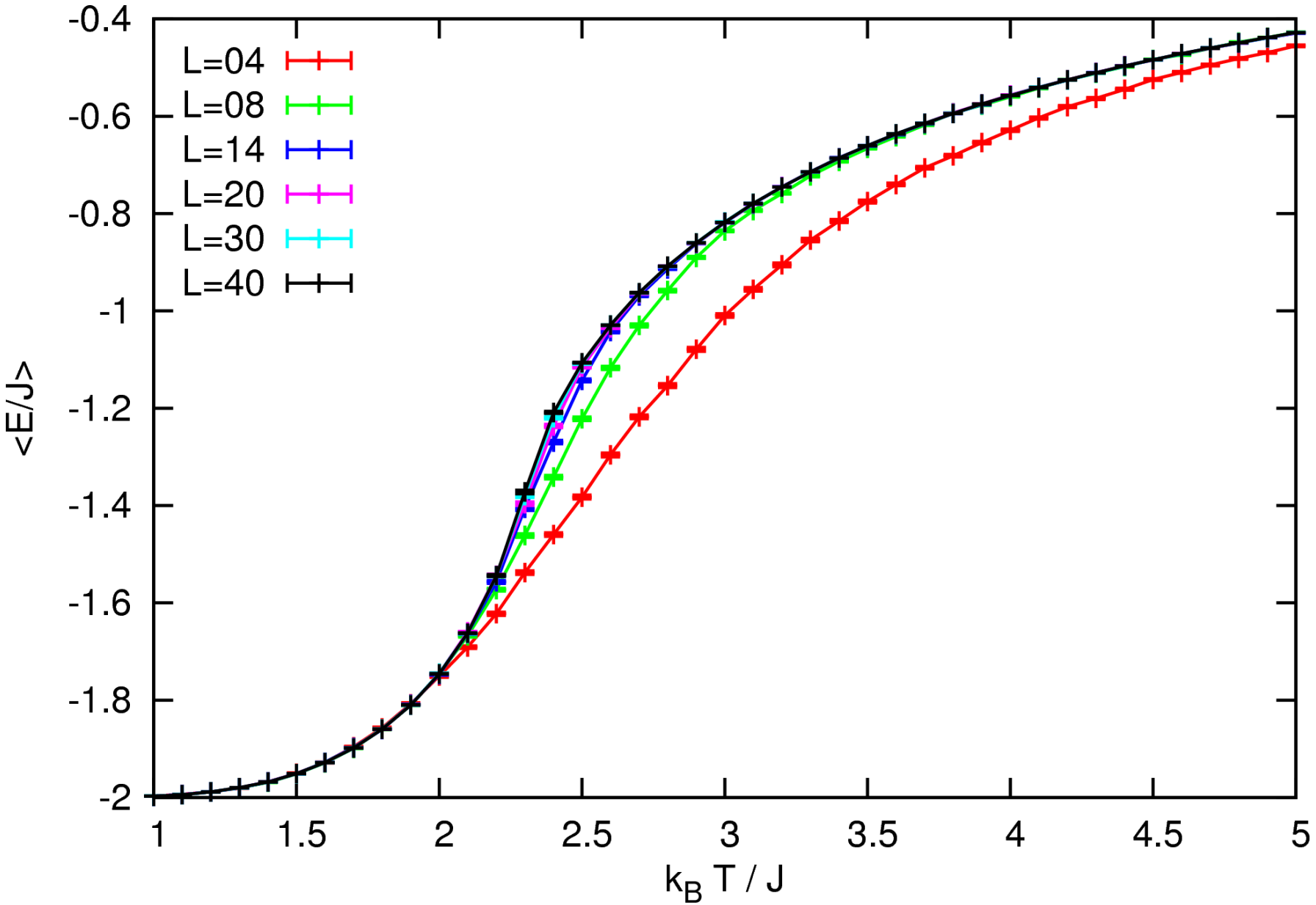
Magnetization (10^5 sweeps)



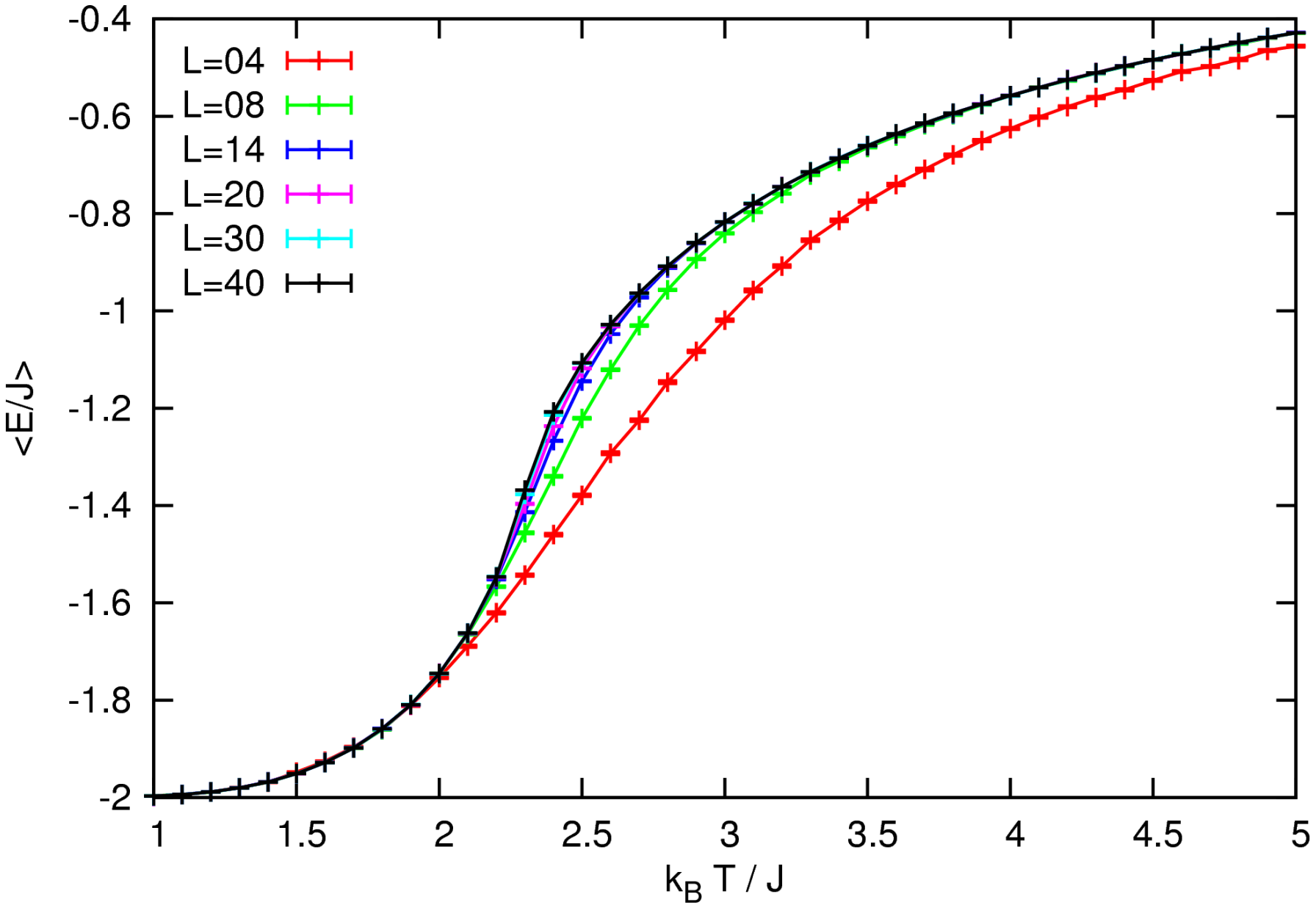
Magnetization (10^5 sweeps) - Wolff update



Energy (10^5 sweeps)



Energy (10^5 sweeps) - Wolff update



Smart observables: susceptibility, specific heat & Binder's cumulant

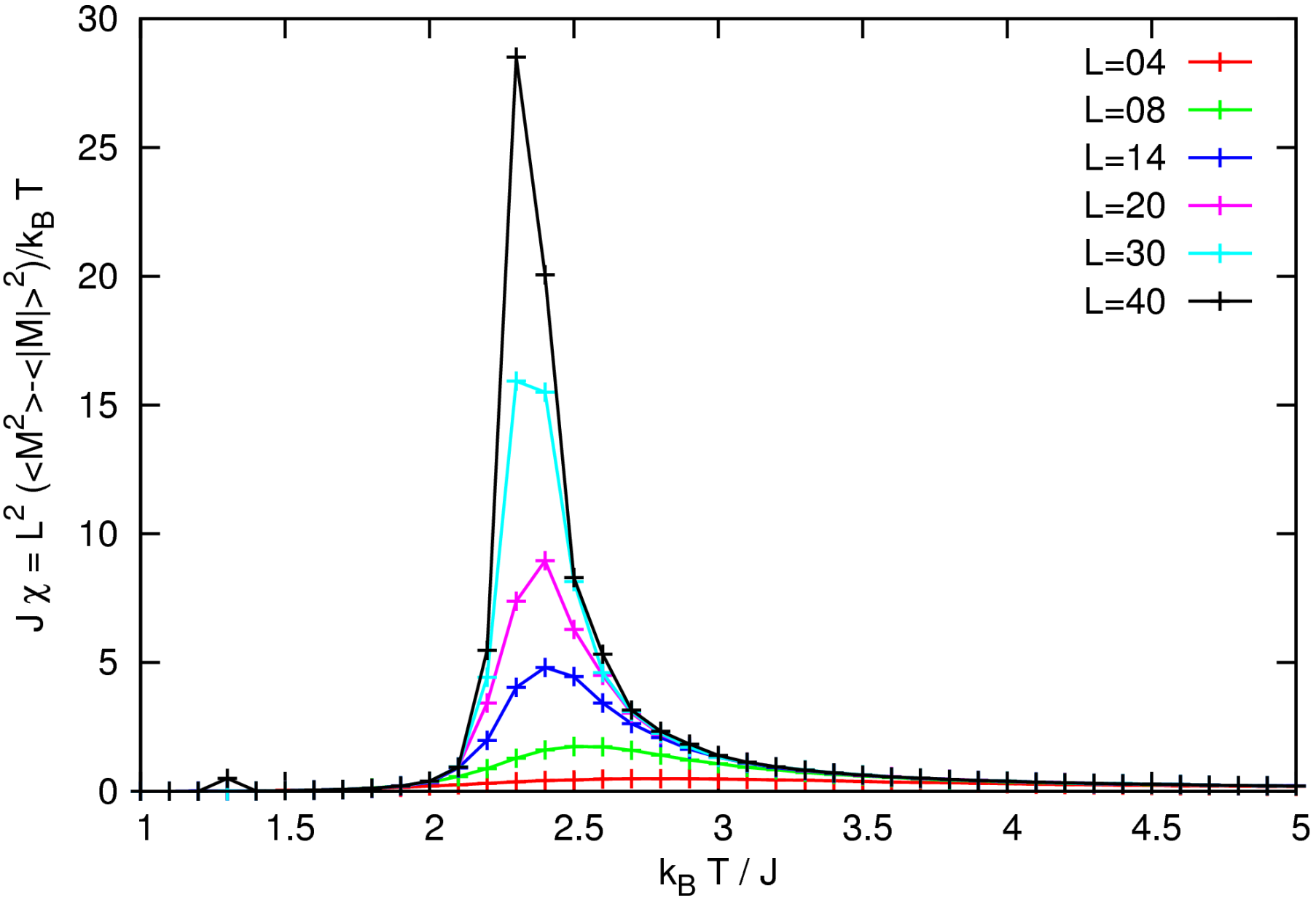
Now we turn to observables which should be particularly suited for determining the Curie temperature T_c.

Magnetic susceptibility: $k_B T \chi = \langle M^2 \rangle - \langle M \rangle^2$

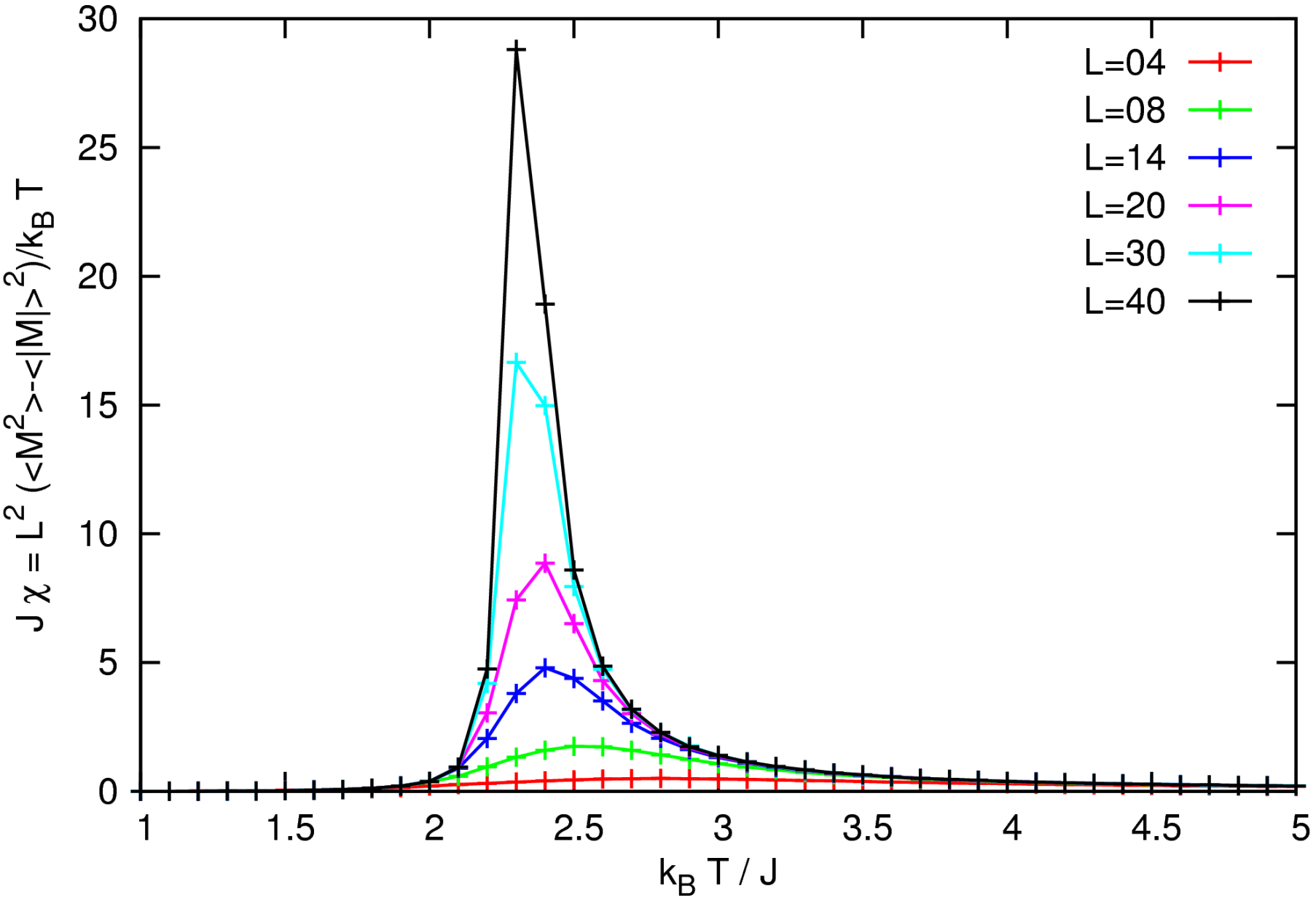
Specific heat: $k_B T^2 C_V = \langle H^2 \rangle - \langle H \rangle^2$

For the moment, only the data is used, no additional input like critical exponents.

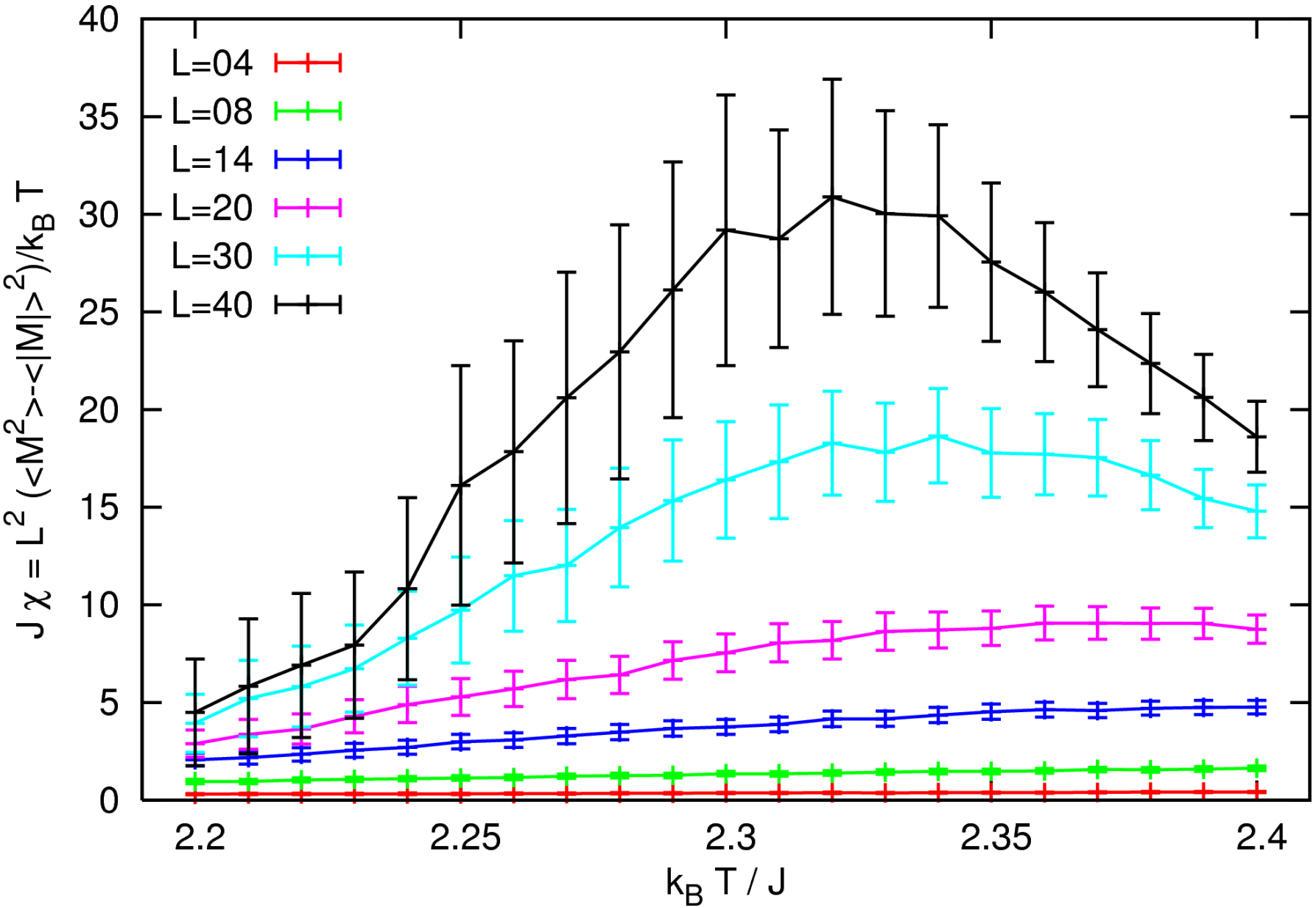
Magnetic susceptibility (10^5 sweeps)



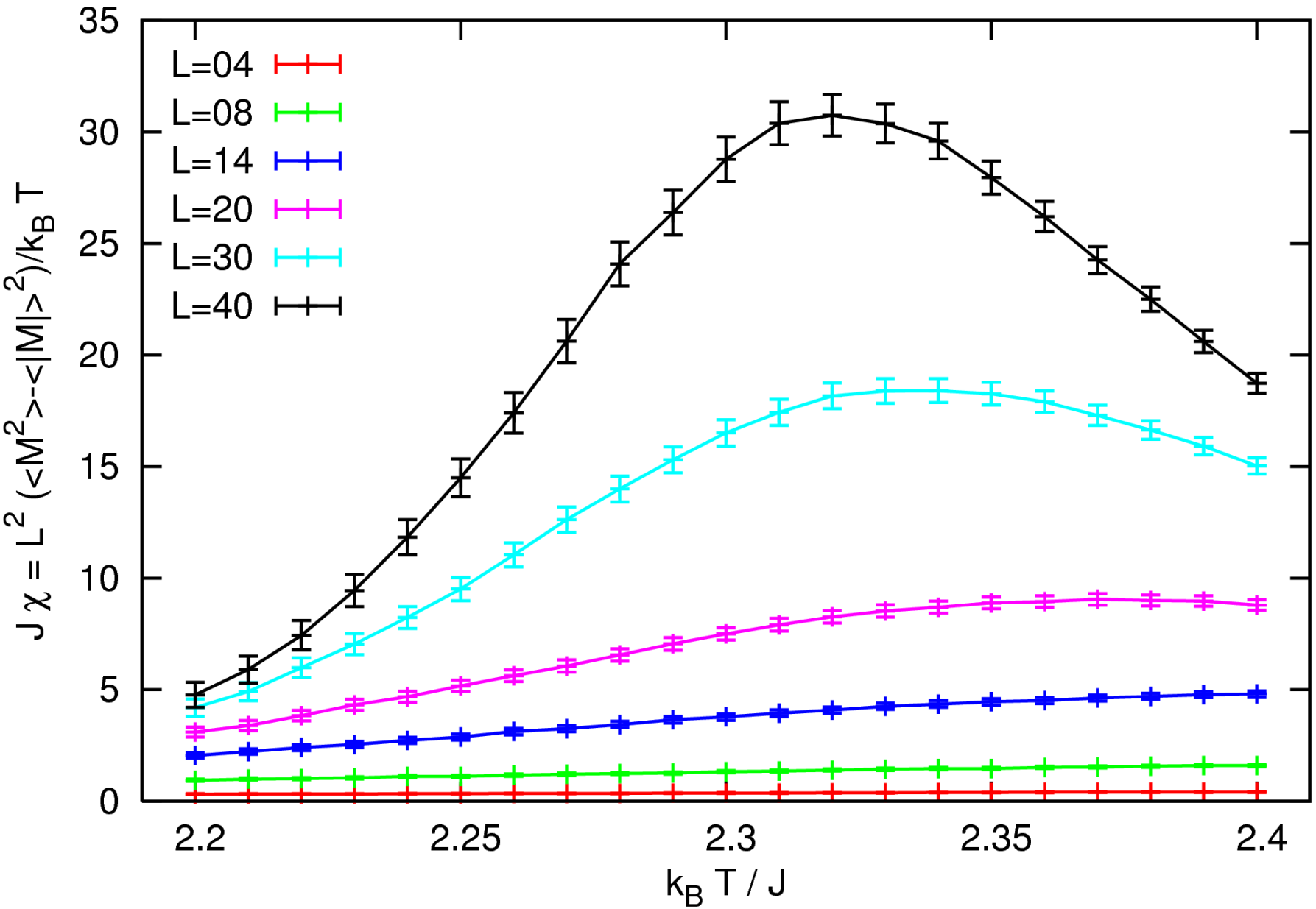
Magnetic susceptibility (10^5 sweeps) - Wolff update



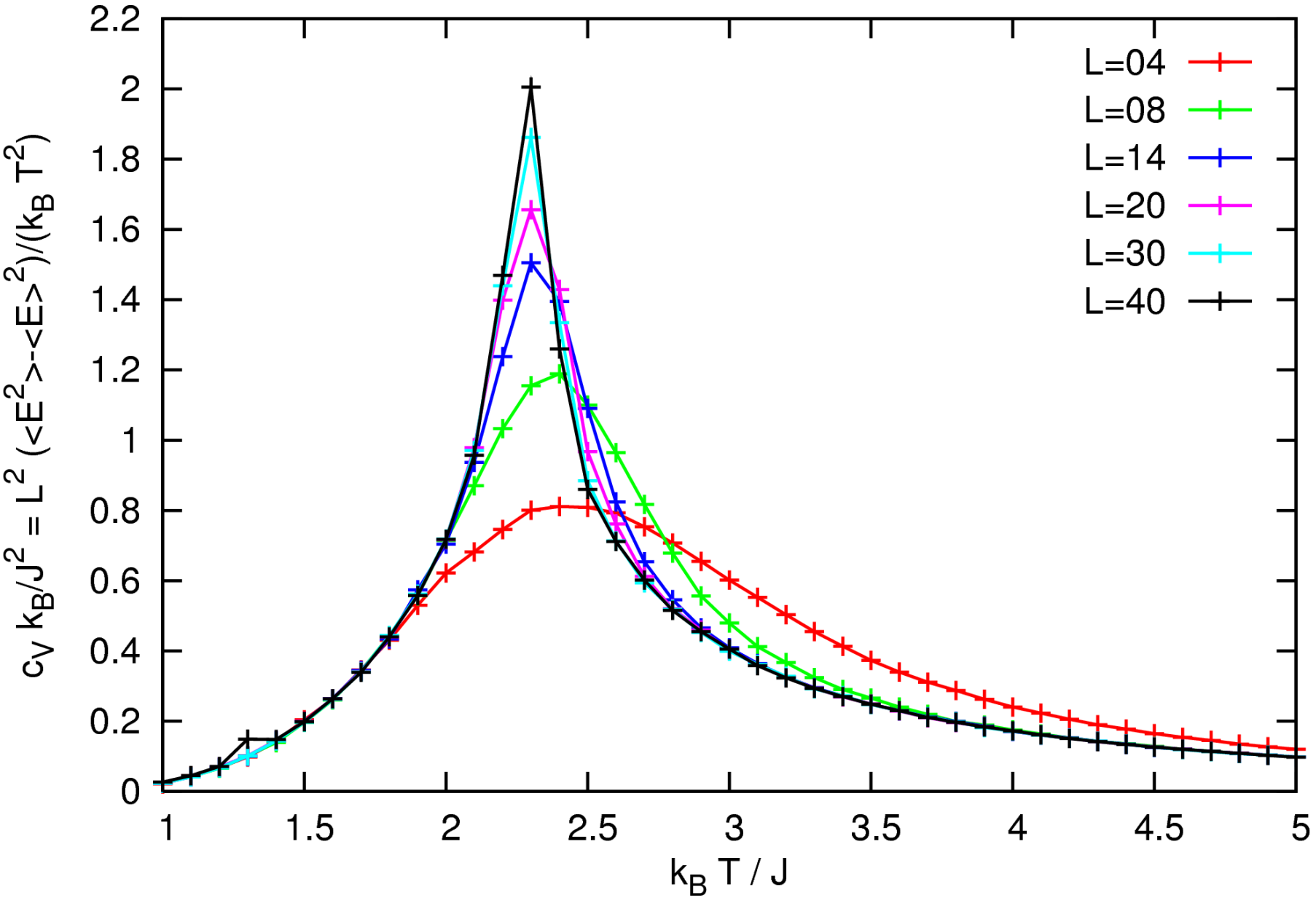
Magnetic susceptibility near T_c (10^6 sweeps)



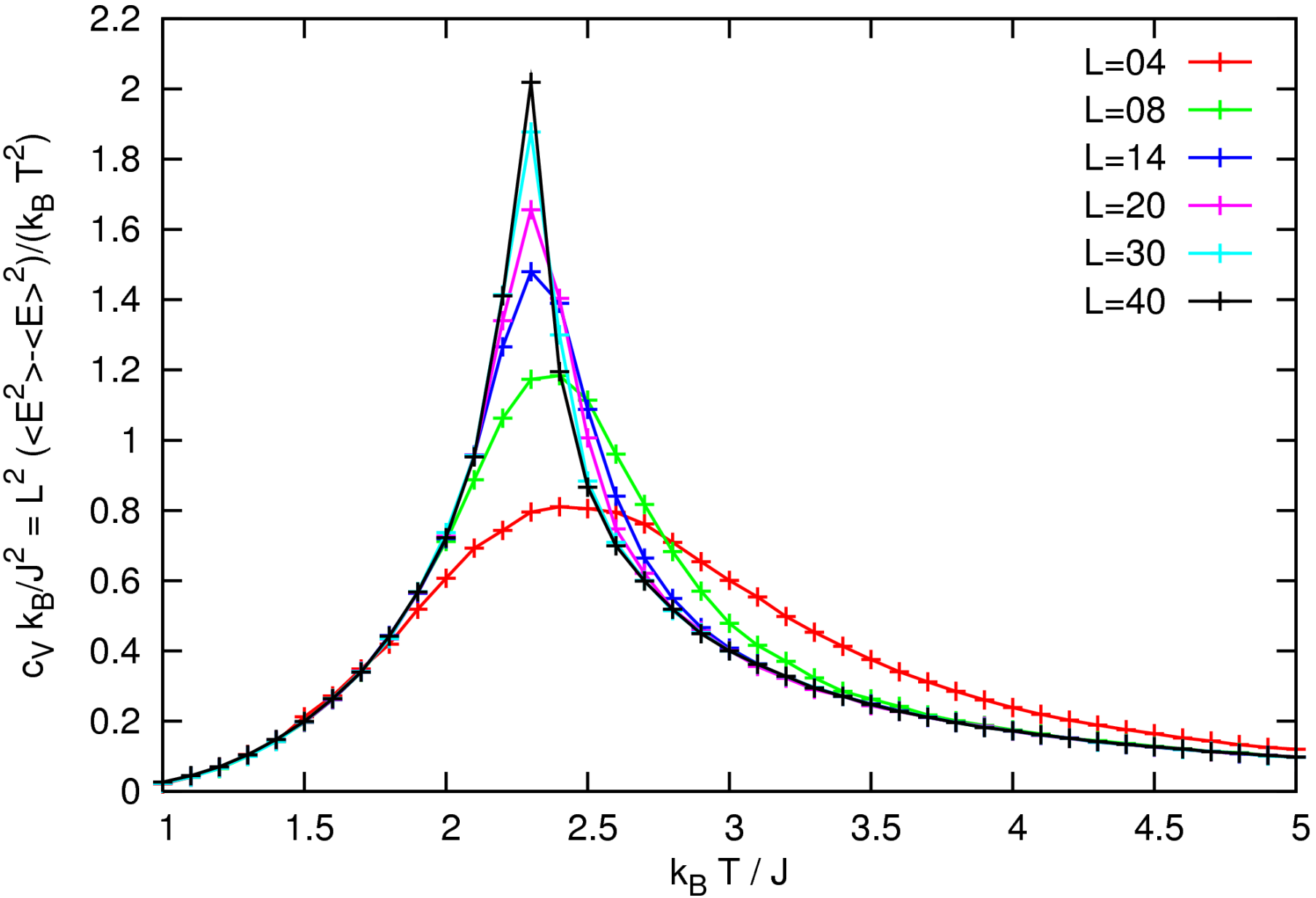
Magnetic susceptibility near T_c (10^6 sweeps) - Wolff update



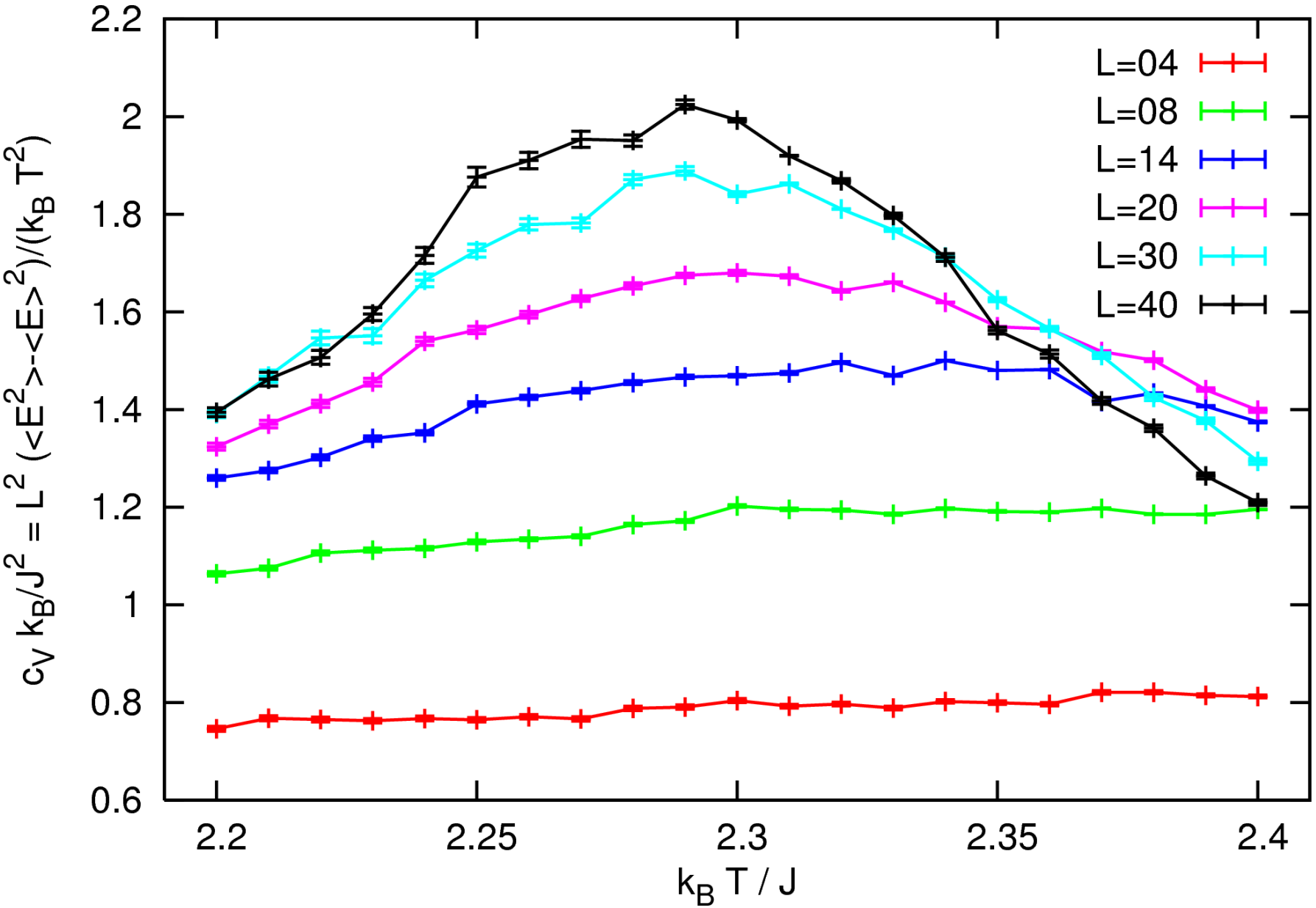
Specific heat (10^5 sweeps)



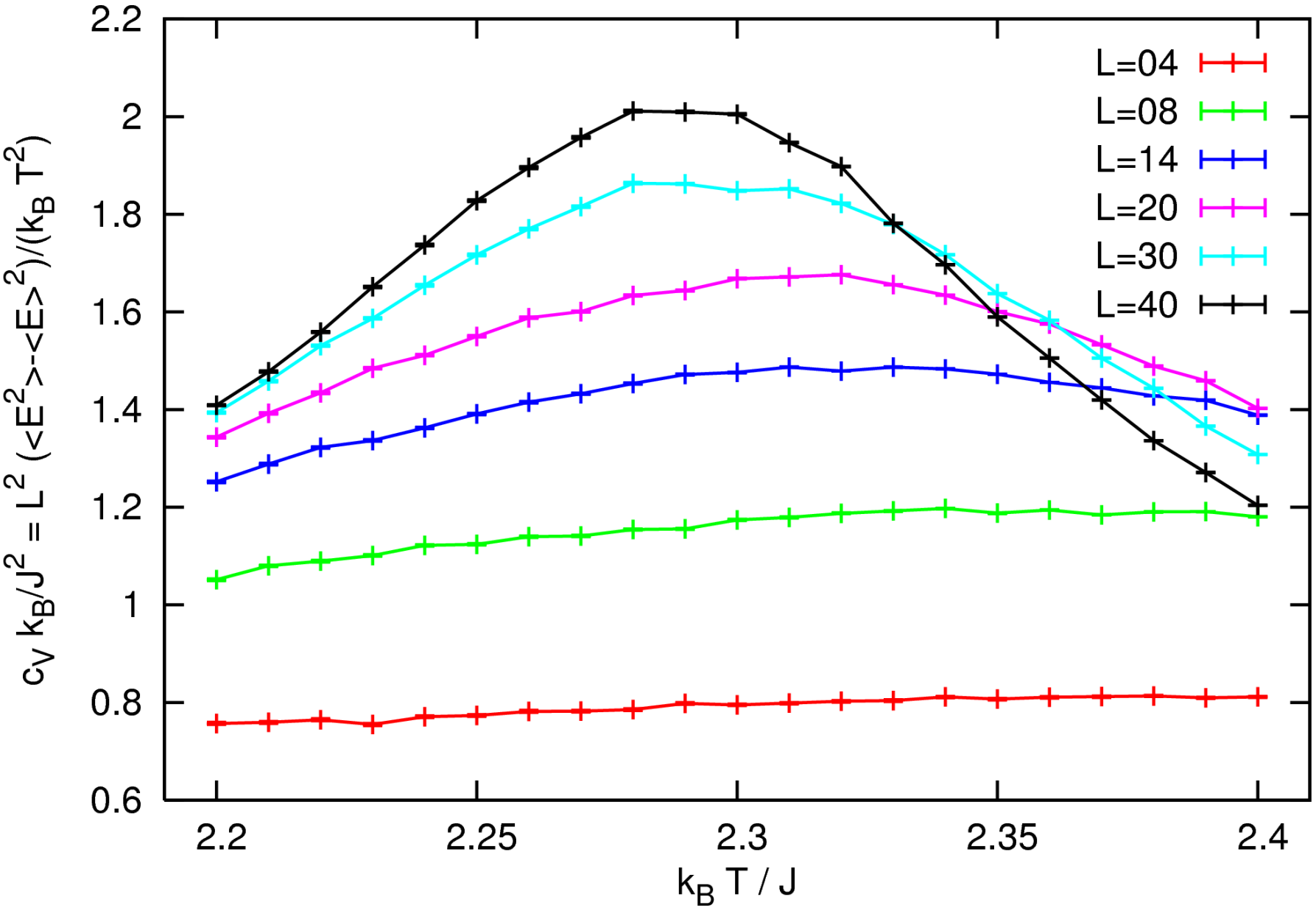
Specific heat (10^5 sweeps) - Wolff update



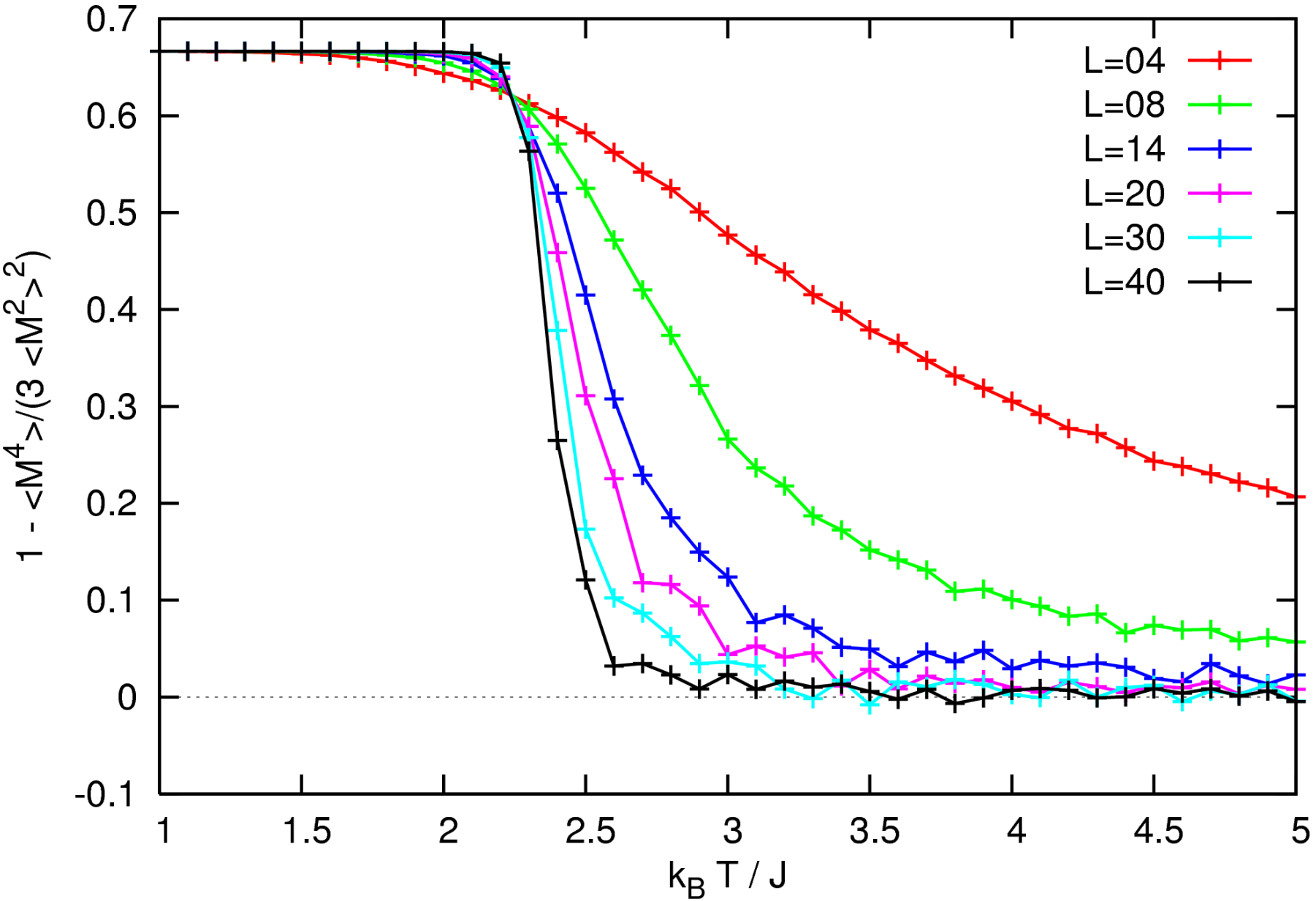
Specific heat near T_c (10^6 sweeps)



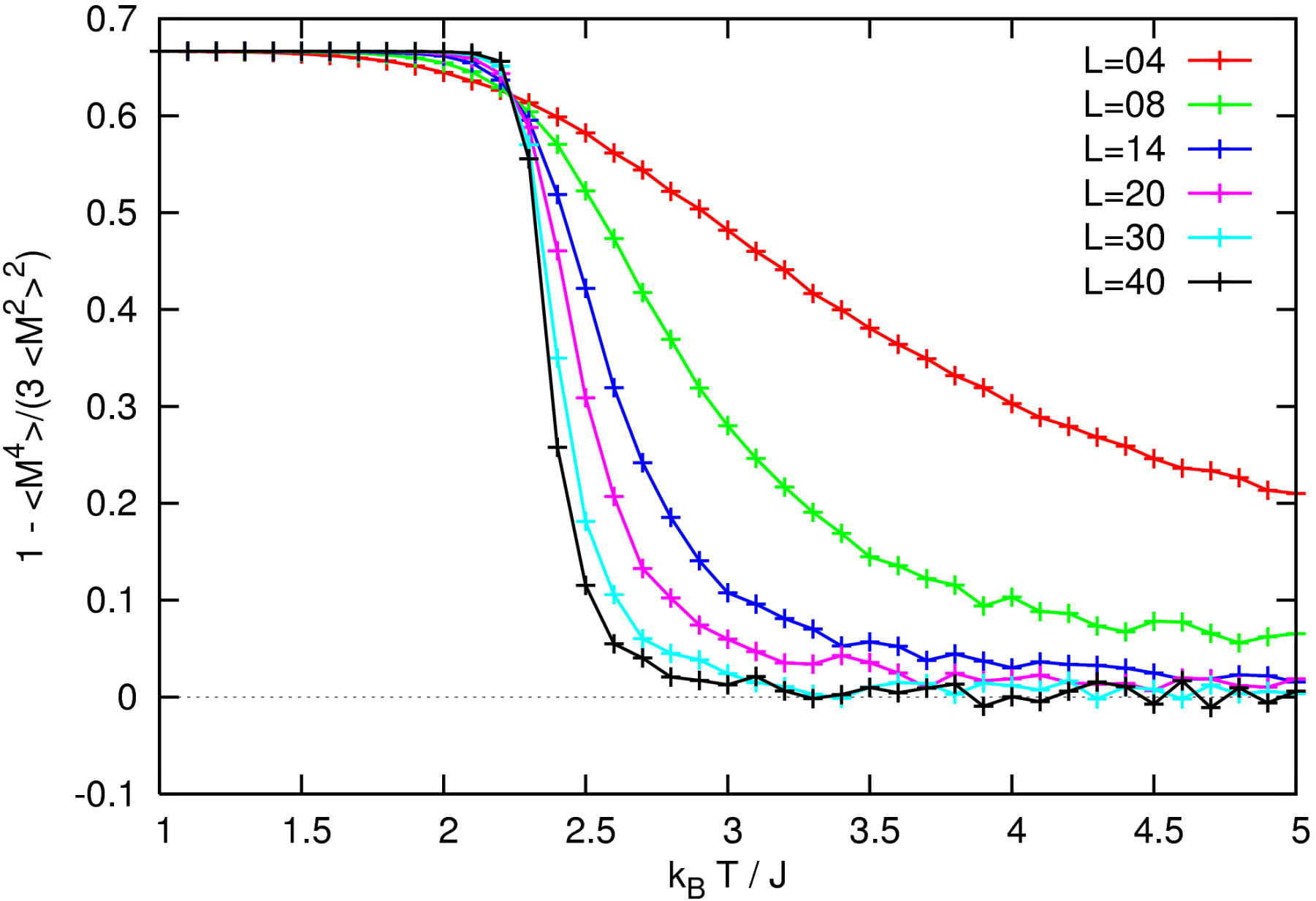
Specific heat near T_c (10^6 sweeps)g - Wolff update



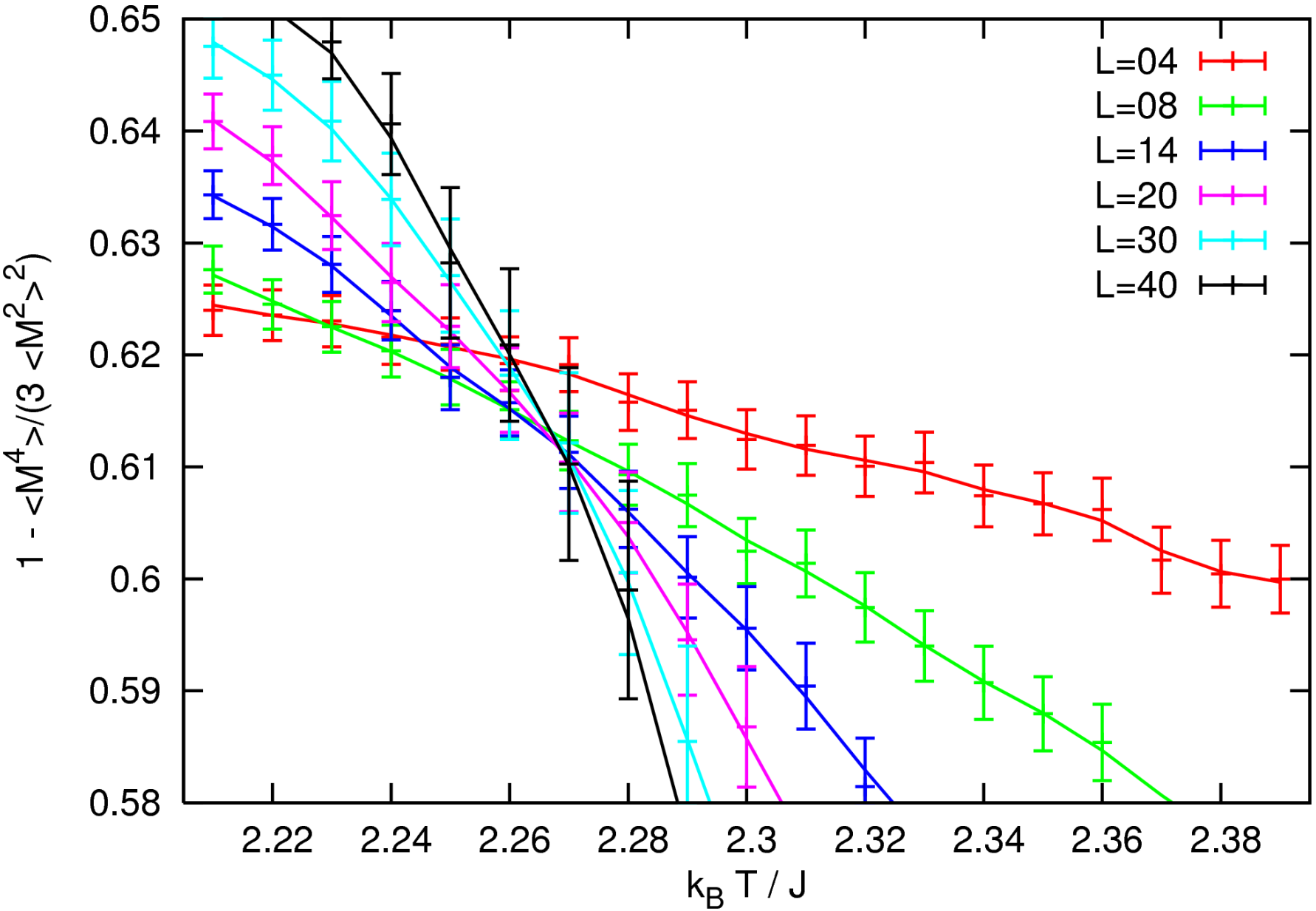
Binder's cumulant (10^5 sweeps)



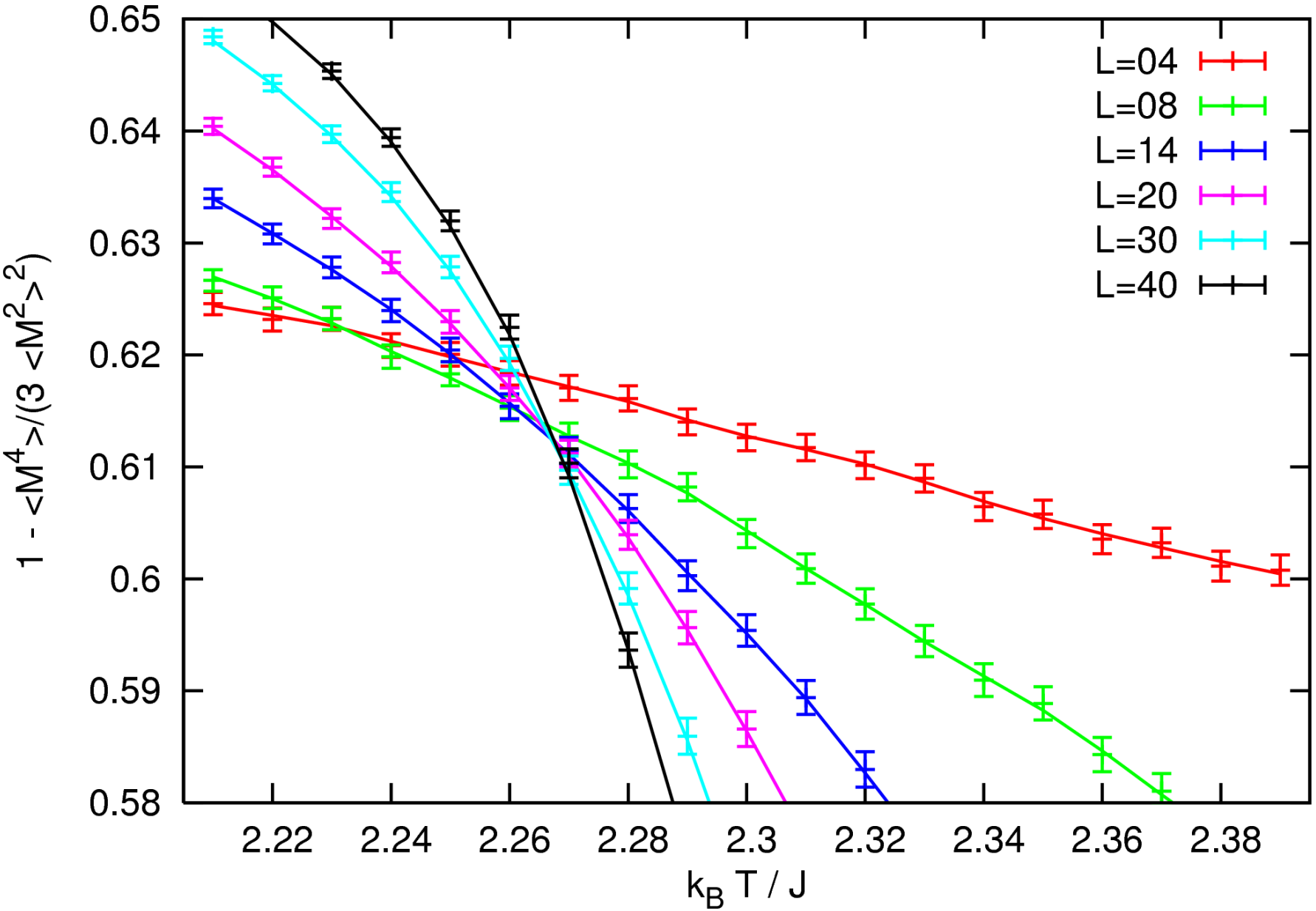
Binder's cumulant (10^5 sweeps) - Wolff update



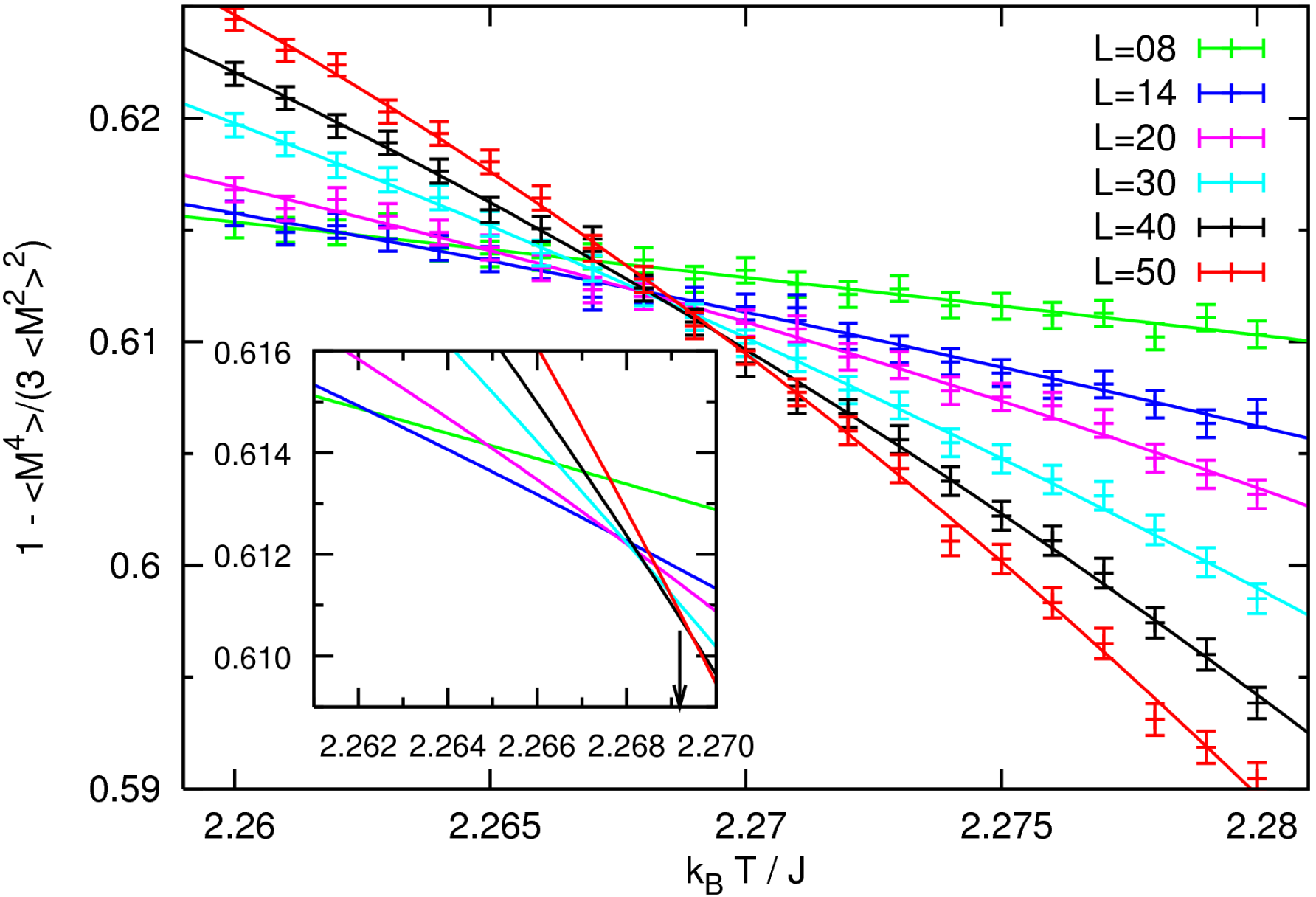
Binder's cumulant near T_c (10^6 sweeps)



Binder's cumulant near T_c (10^6 sweeps) - Wolff update



Binder's cumulant near T_c (10^6 sweeps) - Wolff update



Scaled magnetization near T_c (10^6 sweeps) - Wolff update