Introduction

Solid state theory aims at a description of intrinsic properties of solid materials by finding and extracting information from suitable models. A good model is complicated enough to capture interesting aspects of nature, but simple enough to be solvable (at least within some limits or within controlled approximations) and to provide insights about the associated mechanisms. Even if it was possible to construct a theory which exactly predicts all measurable properties of solids, more abstract models would still be necessary in order to classify the different properties and the different classes of physical systems and in order to identify which "ingredients" are important for some particular effect. In this sense, the presence of free parameters in *model Hamiltonian approaches* is not necessarily a shortcoming compared to *ab initio* methods, but a very useful handle for understanding phenomena in a general context. Often, different simplified models give reasonable descriptions for different properties of the same material. For a more realistic description one can then build up a hierarchy of more complex theories, thus broadening the range of validity and improving on the accuracy of agreement to experiment. The main advantage of a *microscopic model* over a phenomenological theory is that its inherent approximations are known so that it can, at least in principle, be made more realistic in a controlled way.

Solids are constituted of a macroscopic number of positively charged atomic nuclei and negatively charged electrons. In single crystals, the heavy nuclei can be assumed to form a rigid periodic lattice. Furthermore, due to the strong Coulomb force, a large fraction of the electrons is typically tightly bound in atomic-like shells around the ions. The resulting core ions then provide a periodic background potential for and shield the interaction between the remaining electrons. These *valence electrons* determine, to leading order, the electronic, magnetic, and thermal properties of solids. In this work, we will exclusively study the electronic system and mostly restrict our treatment to the valence electrons.

Even in the limit of vanishing interaction between the (valence) electrons, their dispersion is split by the lattice potential into an infinite number of energy bands (Bloch, 1928). Due to the Pauli exclusion principle each eigenstate, characterized by its momentum (within the Brillouin zone), band index, and spin, can only be occupied by one electron. Consequently, there is some trivial correlation between the electrons even at the noninteracting level. The (lattice-dependent) energy of noninteracting electrons is commonly referred to as *kinetic energy*. It is the *potential energy* associated with the (shielded) electron-electron *Coulomb interaction* which introduces genuine *correlations* between the valence electrons and makes the problem