taking intersite interactions into account. Apart from just keeping additional terms of (1.9), one may also choose a semi-effective approach including Kondo type interactions, e.g., for modeling manganites. Compounds with several inequivalent ions per unit cell may be modeled directly or by using orbitals on an effective Bravais lattice. Further possible extensions which introduce qualitatively new physics [compared to (1.14)] are disorder, phonons, and spin-orbit interaction. A more detailed discussion (including Hamiltonians and references) can be found in App. A.1. While most of the material provided there is intended to put our work into perspective, we will analyze the implications of (off-diagonal) disorder for magnetic order, the density of states, and transport properties in Sec. 2.2, chapter 3, and subsection 4.4.4 and study a multi-band model in chapter 5. For the moment, however, we will concentrate on the (clean) one-band case for which we can achieve the high accuracy (in the limit of infinite dimensionality, see below) needed for a reliable determination of the low-temperature phase diagram in chapter 3.

1.2 Dynamical Mean-Field Theory

Even though the Hubbard model (1.14) is a drastically simplified model for strongly correlated electron systems, very few exact statements can be made for lattice dimensions $d > 1$ (see above). Therefore, in general, additional approximations have to be made. One particularly successful approximation for the Hubbard model and its extensions is the Dynamical Mean-Field Theory (DMFT), also known as the *limit of infinite dimensions* (Metzner and Vollhardt, 1989). In the following, we will first characterize the DMFT in comparison to other approaches used in the literature and discuss its unique advantages in the context of the paramagnetic metal-insulator transition (MIT) studied in chapter 3. We will then, in turn, explore the implications of a large coordination number Z for spin models, noninteracting fermions, and diagrammatic perturbation expansions for the Hubbard model and see that (for appropriate scaling) the Hubbard model indeed remains nontrivial. It can be mapped to a single impurity Anderson model (SIAM) plus a self-consistency condition in the limit $Z \to \infty$. Possible extensions of the theory towards lower dimensions are referred to App. A.1.

Approximate analytic methods are only reliable if they are controlled by a parameter which becomes small in some limit of the problem. Typical examples are weak-coupling perturbation theories in U/t of first order, i.e., Hartree-Fock theory (HF), a *static* mean-field approximation (Penn, 1966), or of second order (Georges and Yedidia, 1991; van Dongen, 1994*a*). Further examples are strong-coupling expansions in t/U (Harris and Lange, 1967; Takahashi, 1977; van Dongen, 1994*b*), and expansions for high temperature or for low density n, i.e., the T-matrix approximation for $d \geq 2$ (Abrikosov and Khalatnikov, 1958; Galitskii, 1958; Landau, 1959; Engelbrecht, Randeria, and Zhang, 1992).

The DMFT is a controlled approach in the above sense; its small parameter is the inverse coordination number $1/Z$. It is a conserving approximation which guarantees thermodynamic consistency. Due to its nonperturbative character, the DMFT is