Little is usually said in the literature about the distribution of nongeneric momenta and the topology of Fermi surfaces in high dimensions. Even Gebhard's (1997) motivation for the random dispersion approximation (RDA, see below) which is based on the generic factorization (1.24) of the 2-particle DOS is somewhat vague and partially incorrect in this respect. In App. A.2, we attempt a more careful characterization of generic and nongeneric momenta. While the modifications relative to Gebhard's view do not explain the discrepancy between RDA result and all recent DMFT results for the MIT (see chapter 3), they shed more light on foundations of the DMFT which will prove useful for our development of a generalized dispersion formalism in Sec. 2.3.

1.2.3 Simplifications for the Hubbard Model in $Z \to \infty$

Up to now we have concentrated on the noninteracting tight-binding part of the Hubbard model Hamiltonian, since the local Coulomb interaction remains unscaled for $Z \to \infty$. We have seen that the dependence of the noninteracting 2-particle DOS (1.22) on the momentum transfer $\boldsymbol{q}$ becomes essentially irrelevant in this limit (i.e., for generic $\boldsymbol{q}$). This observation can be generalized to higher order correlation functions (see Sec. 4.3). The connection to the interacting problem is made by the real-space diagrammatic perturbation expansion where properties of the full interacting system such as its single-particle Green function $G_{ij,\sigma}$ are expressed in terms of bare (noninteracting) Green functions $G^0_{ij,\sigma}$ and interactions. For the Hubbard model with only local interactions, interaction lines reduce to local 4-leg vertices.[15]

Let us show by power counting that for $Z = \infty$ all pairs of vertices i and j must coincide when they are connected by at least three independent lines: since the hopping is scaled as $t = t^*/\sqrt{Z}$, each (direct) path made of fermion propagators connecting i to j involves a factor of order $Z^{-\|i-j\|/2}$. For fixed site i, we have at most $Z^{\|i-j\|}$ choices for a "first" path connecting i to j or a topologically equivalent site. In all regular lattices, this first path selects $\|i-j\|$ directions. Since j is now fixed, the freedom of choice for the second and additional paths connecting i and j is generically reduced to a permutation of these directions which introduces additional prefactors independent of Z. For a total of P_{ij} independent paths the scaling factor[16] is then $\mathcal{O}(Z^{\|i-j\|(1-P_{ij}/2)})$ and vanishes in high dimensions for $P_{ij} \geq 3$.[17] Thus, the proper self-energy becomes local for $Z \to \infty$,

$$\Sigma_{ij,\sigma}(\omega) \stackrel{Z\to\infty}{\longrightarrow} \delta_{ij}\Sigma_\sigma(\omega), \tag{1.25}$$

which can be directly used to eliminate one summation over lattice sites in the fol-

[15] NN interactions of the density-density type [cf. (A.7)] reduce to their Hartree contribution and can, thus, be easily included in all computations (Müller-Hartmann, 1989*a*).

[16] Obviously, these considerations do not directly apply to the fully frustrated model with infinite range hopping which has been studied numerically in the disordered case (Georges, Kotliar, and Krauth, 1993) and been analytically solved in the clean case (van Dongen and Vollhardt, 1989).

[17] Note that this argument relies on the dimension being much larger than P_{ij} and, ultimately, larger than the order of the diagram. Conversely, a generalization to finite dimensions would have to stop at finite-order perturbation theory (cf. footnote 5 on page 30).