be performed analytically for the semi-elliptic Bethe DOS (see Sec. 2.2) used in chapter 3. In contrast, the solution of the impurity problem (1.31) is highly nontrivial.[21] Most numerical methods developed for the treatment of SIAMs with fixed bath could be adapted to the DMFT problem, e.g., solutions based on exact diagonalization (ED) (Caffarel and Krauth, 1994; Georges et al., 1996), the noncrossing approximation (NCA) (Keiter and Kimball, 1970; Bickers, Cox, and Wilkins, 1987; Pruschke and Grewe, 1989; Pruschke, Cox, and Jarrell, 1993), the fluctuation-exchange approximation (FLEX) (Bickers, Scalapino, and White, 1989; Bickers and Scalapino, 1989; Bickers and White, 1991), the numerical renormalization group (NRG) (Wilson, 1975; Krishna-murthy, Wilkins, and Wilson, 1980; Costi, Hewson, and Zlatić, 1994; Bulla, 2000), and quantum Monte Carlo (QMC) algorithms. We will introduce the QMC method in Sec. 1.3 and use it throughout the numerical parts of this work; we will later also compare to results obtained from the other methods mentioned above as well as from iterated perturbation theory (IPT), which for half-filling ($n = 1$) is based on the following second-order approximation for the self-energy (Georges and Kotliar, 1992),

$$\Sigma^{\mathrm{IPT}}_{\sigma n} = \frac{U}{2} + U^2 \int_0^\beta \mathrm{d}\tau\, \mathrm{e}^{i\omega_n \tau} \big(\tilde{\mathcal{G}}_\sigma(\tau)\big)^3; \quad \tilde{\mathcal{G}}_\sigma(\tau) = \mathcal{G}_\sigma(\tau) - \frac{U}{2} \tag{1.36}$$

and becomes exact both at weak and strong coupling (Zhang, Rozenberg, and Kotliar, 1993).

While the DMFT is a very successful theory for correlated electron systems due to its nonperturbative character, it misses some important physics even in three dimensions and may fail completely for two dimensional systems. In particular, phenomena which involve strong momentum dependence such as d-wave superconductivity cannot be described within the DMFT. Attempts to build up a more widely applicable theory by systematically including $\mathcal{O}(1/Z)$ corrections have failed so far to produce breakthroughs for finite-dimensional systems (Gebhard, 1990; Vlaming and Vollhardt, 1992; van Dongen, 1994*c*; Schiller and Ingersent, 1995; Pruschke, Metzner, and Vollhardt, 2001).[22] However, significant progress has been made recently using schemes which interpolate between the DMFT single impurity problem and finite-dimensional clusters (with periodic boundary conditions). These fascinating extensions of the DMFT, namely the DCA and the CDMFT are reviewed in App. A.3, together with the RDA which was constructed as an alternative to the DMFT self-consistency approach.

[21] An exception is the application of (1.30) and (1.31) to a Lorentzian DOS $\rho(\epsilon) = t/(\pi(\epsilon^2 + t^2))$ which can be realized on lattices with long range hopping (Georges and Kotliar, 1992). For this DOS (which is clearly pathological due to its infinite variance), the Weiss function is independent of U; furthermore (1.31) is solvable by Bethe ansatz in this case so that many properties can be obtained analytically.

[22] Recently, an improved version of the Schiller-Ingersent scheme was applied for the hypercubic lattice using the FLEX approximation. Although no violation of causality was observed in this case, the causality of the method in general is still unclear (Zaránd, Cox, and Schiller, 2000).