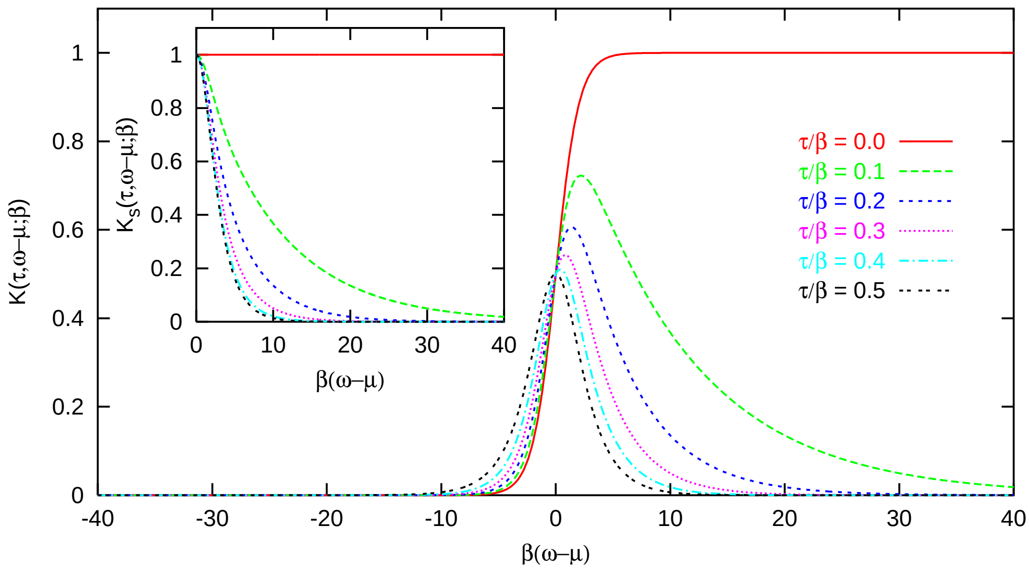

Figure 1.2: General fermion kernel (1.56) for an equidistant set of values of $\tau/\beta \leq 1/2$. Except for $\tau = 0$, large frequencies are suppressed exponentially. Inset: symmetrized fermion kernel (1.57).

values of τ at large absolute frequencies $|\omega|$ as illustrated in Fig. 1.2. For a symmetric problem, i.e., symmetric noninteracting DOS and $n = 1$, the integral in (1.55) can be restricted to positive frequencies when the symmetrized fermion kernel

$$K_s(\tau, \omega; \beta) = K(\tau, \omega; \beta) + K(\tau, -\omega; \beta) \tag{1.57}$$

shown in the inset of Fig. 1.2 is used. We stress that a very small number $\Lambda = \mathcal{O}(10)$ of time slices poses a more serious limitation for obtaining reliable spectra with good resolution than the exponential nature of the kernel since the number of degrees of freedom which can be reliably resolved in a spectrum is obviously much smaller than the number of data points $\{G(l\Delta\tau)\}$ and since there always exists an infinite number of spectra which correspond to the same data. Still, in general, the resolution is much better at smaller frequencies and, according to Nyquist's theorem, essentially no information can be obtained from QMC for $\omega > \Lambda\pi/\beta = \pi/(\Delta\tau)$. Before we address the full analytic continuation problem and introduce the maximum entropy method, we collect some useful relations (denoting the m^{th} derivative as $G^{(m)}$),

$$G(\beta) = n, \qquad G(0^+) = 1 - n, \tag{1.58}$$

$$G^{(m)}(0) + G^{(m)}(\beta) = (-1)^m \langle (\omega - \mu)^m \rangle_{A(\omega)} \tag{1.59}$$

$$G(\beta/2) \approx \frac{\pi}{\beta} \overline{A(\omega)}\Big|_{|\omega-\mu| \lesssim \pi/\beta}. \tag{1.60}$$

Since the filling given by (1.58) is known in the symmetric case, the value of $G(0)$ then provides no useful information which is also seen in the inset of Fig. 1.2. Equation (1.59) also shows the loss of high-frequency information from the discretization of