which is the essential lattice input to the optical conductivity within the bubble approximation (see chapter 4 where results for $\tilde{\rho}(\epsilon)$ are also presented). However, a fast and reliable way of computing the DOS is always useful since in spite of considerable analytic effort a numerical integration is unavoidable even for the hypercubic lattice with NN hopping in $d \geq 2$ [see, e.g., Schlipf (1998)].

The most important results of this chapter are the exact analytic expression and numerical estimates for extended hopping matrix elements defined on a hypercubic lattice which yield a semi-elliptic DOS in the limit $d \to \infty$. In fact, the new method developed in this chapter for determining dispersion, transport properties, and matrix elements in $d = \infty$ is very general; it can be applied for any arbitrary target DOS. The method is based on a generalization of the observation that the dispersion of the fcc lattice can be written as a function of the dispersion of the hc lattice. This expression, a (quadratic) polynomial in $\epsilon_{\boldsymbol{k}}^{\text{hc}}$, implies a simple transformation rule for the DOS. In the course of this thesis it became apparent that parts of the necessary formalism had already been worked out, but had not been published. Extending van Dongen's (2001) construction via expansion of the dispersion in Hermite polynomials we will show that both the hopping matrix elements and all information (e.g., the Fermi velocity) needed for the computation of transport properties can be obtained in a controlled scheme.

The structure of the remainder of this chapter is as follows: In Sec. 2.1 we will first discuss hypercubic lattices with NN hopping matrix elements only, then consider t-t' hopping to NNs and NNNs and, finally, we will treat the fcc lattice which is equivalent to a hc lattice with NNN hopping only. Conventional ways of constructing a semi-elliptic DOS in $d = \infty$, i.e., via the fractal Bethe lattice and the disordered infinite-range hopping model are summarized in Sec. 2.2. The new general dispersion scheme is then developed in Sec. 2.3. As a first practical application we use this method for the construction of a regular lattice with Bethe semi-elliptic DOS and begin with the characterization of its transport properties in Sec. 2.4. The robustness of the new method is demonstrated by studying both the effects of truncation of the hopping range and of finite dimensionality. Our treatment of the hyperdiamond lattice, a regular non-Bravais lattice with a reduced number of loops, in App. B supplements the findings of this chapter and our discussion of loops in subsection 4.4.2.

2.1 Hypercubic Lattice and Extensions

2.1.1 Definitions and Analytical Considerations

The d-dimensional hypercubic (hc) lattice is spanned by d orthonormal primitive vectors (setting the lattice spacing to $a = 1$) so that the cartesian components of each lattice vector are integers and the Euclidean length (or 2-norm) of a vector is given by

$$|\boldsymbol{v}| := \|\boldsymbol{v}\|_2 = \Big(\sum_{\alpha=1}^{d} v_\alpha^2\Big)^{1/2} \tag{2.3}$$