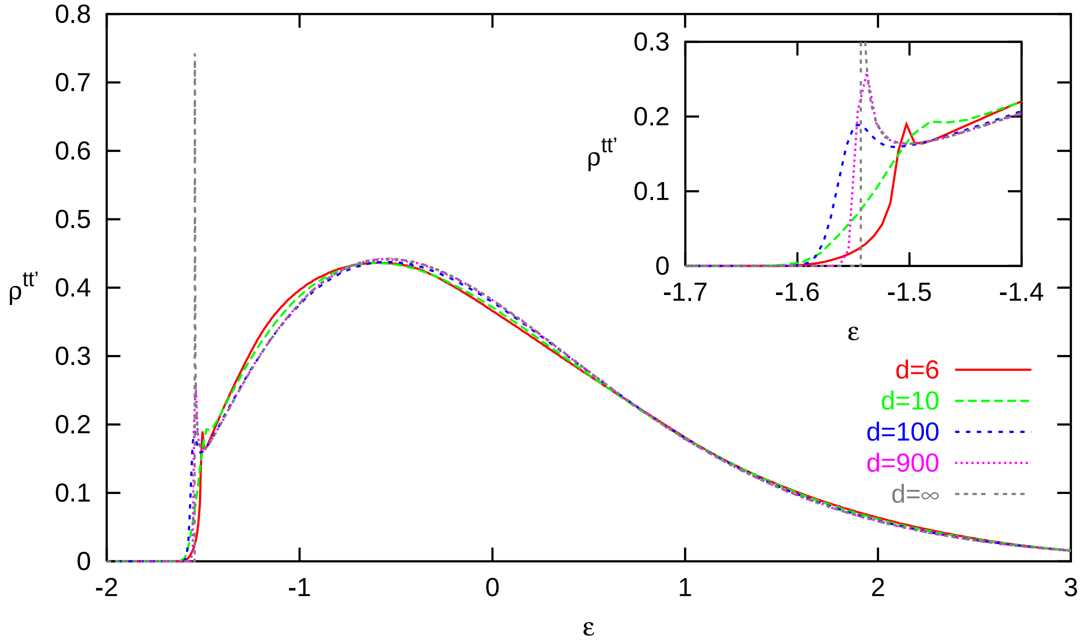

Figure 2.4: DOS of the hypercubic lattice with NN and NNN hopping, $a^* = -0.25$ for high dimensions. As seen in the inset, a sharp singular lower band edge develops for $d \to \infty$.

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picture changes, however, when d is further increased. As shown in Fig. 2.4, for $d \geq 6$ a maximum appears[8] near the lower band edge which develops into a sharp band edge with a square-root singularity (i.e., with the asymptotic form of an inverse square root) for $d \to \infty$. Increasing the absolute value of a^* increases these tendencies as depicted for $a^* = -0.3333$ in Fig. 2.5 and Fig. 2.6. Here, a broad maximum at low energies develops already for $d = 2$ which becomes narrow and distinguishable from the main maximum for $d \approx 10$ and approaches a square-root singularity with a height substantially exceeding the main maximum for $d \gtrsim 100$. We observe that for finite dimensions there is a low energy tail of relative width $1/\sqrt{d}$ which only vanishes in $d = \infty$. An application of $d = \infty$ with NNN hopping to $d = 3$ captures the aspect of non-particle-hole symmetric Hamiltonians, but the removal of the tail extending to negative infinite energy comes at the price of an unphysical singularity which is characteristic of very high dimensions $d \gtrsim 100$ only.

Similar observations can be made for the fcc lattice (formally $a^* = -\infty$) where in $d = 3$ one of the Van-Hove singularities is at the lower band edge leading to a logarithmic divergence. Hence, in this case, sharp band edges with singularities exist in $d = 3$ and $d = \infty$. At intermediate dimensionality, e.g., for $d = 4$, $d = 5$, and $d = 6$, however, the overall shape changes little with dimension while for $d > 10$ the

[8]This feature is absent in $d = 6$ for the alternative scaling $t' = t'^*/d$ (no results shown) to which (2.10) converges in the limit $d \to \infty$. Scaling (2.10) is, however, strongly favorable since it ensures the same ratio of NN and NNN contributions to the variance of the energy in all dimensions; see (2.11).