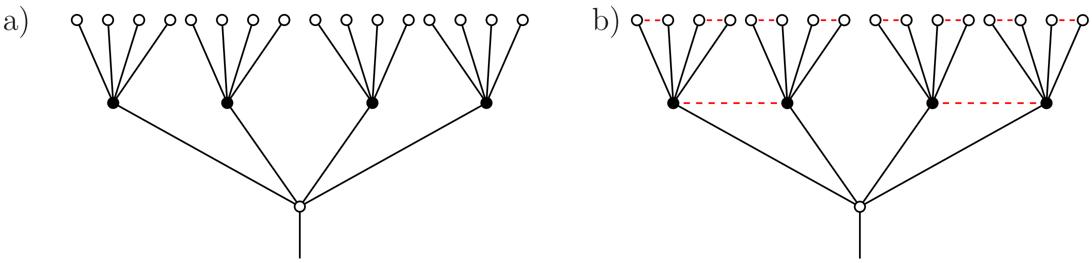

**Figure 2.10: a)** Scheme of Bethe tree for connectivity $K = 4$ (i.e., $Z = 5$). **b)** Scheme of triangular Husimi cactus for connectivity $K = 2$ (i.e., $Z = 6$) which differs locally from the Bethe tree by the presence of a single bond (dashed lines). Black and white dots denote A and B sites on the bipartite Bethe tree, respectively. Corresponding AF order is partially frustrated on the corresponding Husimi tree in finite dimensions.

lattice. In this section, we will take a closer look at the implicit definitions and come to the conclusion that the original construction need not and should not be taken too seriously in the context of applications of the DMFT method to correlated electron systems. We will also consider a Bethe lattice with NNN hopping and derive some more general results from an expansion in terms of self-avoiding loops, the RPE.

### 2.2.1 Bethe Tree, Cayley Tree, and Husimi Cactus

The Bethe lattice of connectivity $K$ is defined as a pseudo-lattice where each site is surrounded by $Z = K + 1$ nearest neighbors and where no closed loops are formed by NN bonds (Economou, 1979) which makes it a tree in the sense of graph theory.[13] While this construct coincides with the usual $d = 1$ lattice for $Z = 2$, it is not a regular lattice for $Z > 2$ as is visualized in Fig. 2.10 a for $K = 4$, i.e., $Z = 5$. Here it is shown as a directed graph where a level number $i \in \mathbb{Z}$ can be assigned to each site, so that a site at level $i$ is connected by one edge to a site in the lower level $i - 1$ and by $K$ edges to sites in the higher level $i + 1$. Its name stems from the fact that the Bethe-Peierls approximation (Bethe, 1935; Peierls, 1936) for crystalline alloys or Ising models becomes exact on this structure (Kurata, Kikuchi, and Watari, 1953).[14] The Bethe tree is often viewed as the infinite-size limit of a Cayley tree which is a finite mathematical tree where all nodes except for the leaves are connected by edges to $Z$ other nodes. The infinite-size limit of the Cayley tree is not unproblematic, since a "spherical" Cayley tree where all leaves have (taxi cab) distance $n$ from a selected central site consists of $N = [Z(Z - 1)^n - 2]/(Z - 2)$ sites of which $N_s =$

[13] A tree is best defined as a connected, planar graph without loops, where a graph is a topological structure which can be represented as a set of points (nodes) and a set of lines (edges) connecting a subset of the nodes. Final segments and the nodes at their ends are called leaves (see, e.g., Weisstein (1998)).

[14] It is easily seen that the Ising model without external field becomes trivial on a lattice without loops by replacing the site variables through (then independent) bond variables $\tau_{ij} = \sigma_i \sigma_j$. Also note that an extension of the Bethe-Peierls approximation to a model with diagonal disorder can be used in order to study Anderson localization (Kawarabayashi, 1993).