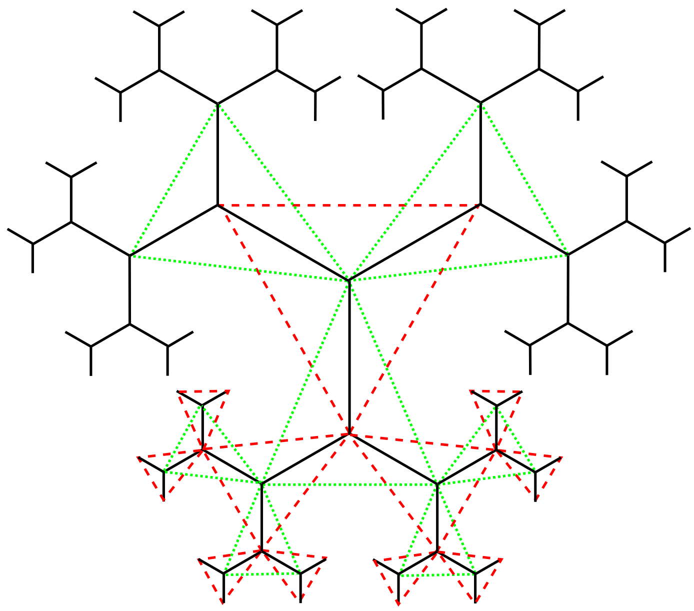

Figure 2.11: Scheme of spherical Cayley tree with connectivity 2 and radius 5 (black solid lines). A selected set of NNN bonds is marked by dashed and dotted lines. Each sublattice formed by the NNN bonds is fully frustrated with respect to AF order.

$Z(Z-1)^{n-1}$ constitute the surface (i.e., are leaves). This is illustrated in Fig. 2.11 schematically for radius $n = 5$ and $Z = 3$. In the limit of infinite radius n, a finite fraction $N_s/N \approx (Z-2)/(Z-1)$ of sites is on the surface (Thorpe, 1981). A tight-binding model defined on such a Cayley tree would be dominated by surface states, in particular in the limit $Z \to \infty$, where $N_s/N \to 1$. This problem can be avoided by using the bulk definition given at the beginning of the paragraph. The Bethe lattice is bipartite; therefore, the generic low-temperature phase of a Hubbard model defined on the Bethe lattice is antiferromagnetic at half filling.

The Husimi cactus is a generalization of the Bethe lattice where each bond is part of exactly one loop and is shown in its triangular form[15] in Fig. 2.10 b. For finite Z, these loops introduce some degree of frustration. Since a single loop becomes irrelevant for $Z \to \infty$, however, all properties of the Husimi cactus converge to that of the Bethe tree in this limit. In contrast, the inclusion of hopping along all NNN bonds (dashed lines in Fig. 2.11) leads to strong magnetic frustration in all dimensions and to an asymmetric DOS (see subsection 2.2.2).

The absence of loops greatly simplifies not only calculations for spin models, but also allows for an exact computation of the Green function for the noninteracting tight-binding model with nearest-neighbor hopping for arbitrary Z. Using a mean-field approach in the spirit of the Bethe-Peierls approximation (Thorpe, 1981) or the renormalized perturbation expansion around the atomic limit (Economou, 1979), one

[15] In the general definition of the Husimi cactus, both the type of polygons, i.e., the length of loops, and the connectivity for each site can be chosen arbitrarily.