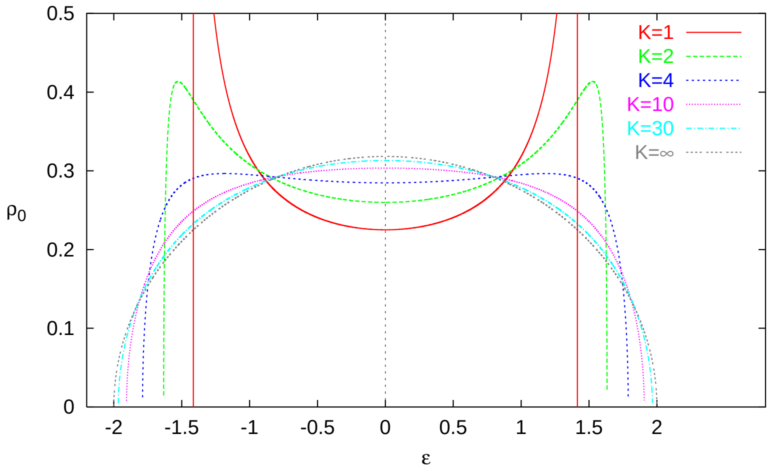

Figure 2.12: DOS (2.23) of the Bethe tree with finite connectivity K. It evolves from the singular form of a chain at $K = 1$ to a semi-elliptic form for $K \to \infty$.

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obtains, e.g., for the diagonal element (hopping amplitude t, on-site energy $\epsilon_0 = 0$),

$$G_{00}(\omega) = \frac{2(Z-1)}{(Z-2)\omega + Z\sqrt{\omega^2 - 4(Z-1)t^2}}, \tag{2.22}$$

where the imaginary part of the square root is taken to be positive for a retarded Green function. The density of states,

$$\rho(\epsilon) = \frac{Z}{2\pi}\frac{\sqrt{4(Z-1)t^2 - \epsilon^2}}{(Zt)^2 - \epsilon^2}, \tag{2.23}$$

is shown in Fig. 2.12 for a hopping amplitude scaled as $t = t^*/\sqrt{Z}$ and $t^* = 1$. The striking feature of the Bethe DOS that the full bandwidth (including all states) is $\mathcal{O}(Z^{1/2})$ instead of $\mathcal{O}(Z)$ (which would be expected from Perron's theorem) can be attributed to the absence of long-wavelength excitations; see Thorpe (1981) and references therein. The spectral weight, concentrated at the band edges for low Z, shifts toward the center with increasing Z until a semi-elliptic form,

$$\rho^{\text{Bethe}}(\epsilon) = \frac{1}{2\pi}\sqrt{4-\epsilon^2}\,. \tag{2.24}$$

is obtained for $Z = \infty$ which we will label as "Bethe DOS" in the remainder of this work.

While the reason for using this Bethe DOS within the DMFT is the presence of sharp band edges, the semi-elliptic form also has the advantage that the $\boldsymbol{k}$ integrated Dyson equation (1.30) can be performed and inverted analytically

$$G(\omega) = \frac{z(\omega)}{2}\left(1 - \sqrt{1 - \frac{4}{z^2(\omega)}}\right), \tag{2.25}$$

$$\Sigma(\omega) = \omega + \mu - G(\omega) - 1/G(\omega)\,, \tag{2.26}$$