$\omega > \max\{|\epsilon_a|, |\epsilon_b|\}$

$$G(\omega) = \int_{-\epsilon_a}^{\epsilon_b} \mathrm{d}\epsilon \, \frac{\rho(\epsilon)}{\omega - \epsilon} = \int_{-\epsilon_a}^{\epsilon_b} \mathrm{d}\epsilon \, \rho(\epsilon) \frac{1}{\omega} \sum_{i=0}^{\infty} \left(\frac{\epsilon}{\omega}\right)^i = \sum_{i=0}^{\infty} \frac{\langle \epsilon^i \rangle}{\omega^{i+1}} \, . \tag{2.28}$$

Thus, the high-frequency behavior of G is easily obtained from moments of the DOS and vice versa. Obviously, a necessary condition for (2.28) and the following conclusions to hold is that all moments $\langle \epsilon^i \rangle$ exist.

The Renormalized Perturbation Expansion (RPE) is an expansion in the hopping matrix elements around the atomic limit where all contributions to the local Green function for some site i can be obtained by considering all decorated self-avoiding loops starting and ending at i. While in finite dimensions the decorations must be computed in terms of special self-avoiding loops which do not touch any site already contained in loops of lower hierarchy, this restriction becomes irrelevant in the limit $Z \to \infty$ where the decorations are nothing but full local Green functions. Thus, for the simplest case of full translational symmetry one arrives at a single closed self-consistency equation for some site i,

$$\begin{aligned} G^{-1}(\omega) &= \omega - \left(G \sum_j t_{ij}^2 + G^2 \sum_{j\neq k} t_{ij} t_{jk} t_{ki} + G^3 \sum_{j\neq k\neq l; j\neq l} t_{ij} t_{jk} t_{kl} t_{li} + \dots \right) && (2.29) \\ &=: \omega - \sum_{n=2}^{\infty} a_n G^{n-1} \, , && (2.30) \end{aligned}$$

where the coefficients a_n measure the weight of self-avoiding loops of length n. Note that when all nonzero hopping matrix elements are equal, the simpler expression $a_n = t^n N_n$ applies, where N_n is the number of self-avoiding loops of length n originating from site i which implies a one-to-one correspondence between the DOS and the topology of the lattice.

Multiplying (2.30) by G and inserting the expansion (2.28) we can compute moments of the DOS from the hopping weights order by order

$$\langle \epsilon^0 \rangle = 1\, ; \qquad \langle \epsilon^1 \rangle = 0\, ; \qquad \langle \epsilon^2 \rangle = a_2 \, . \tag{2.31}$$

Enforcing, for simplicity, unit variance by setting the weight a_2 of 2-loops to 1, the higher order moments read

$$\begin{aligned} \langle \epsilon^2 \rangle &= 1\, ; \qquad \langle \epsilon^3 \rangle = a_3 && (2.32) \\ \langle \epsilon^4 \rangle &= 2 + a_4\, ; \quad \langle \epsilon^5 \rangle = 2 + 3a_3 + a_5 \, . && (2.33) \end{aligned}$$

It is easy to check that only (2.33) and corresponding higher order expressions are specific to $d = \infty$, while (2.32) also applies to finite dimensions: the third moment of any DOS is directly determined only by the weight of self-avoiding 3-loops (i.e., the scaled number of 3-loops if all nonvanishing matrix elements are equal for a translation invariant lattice, $\langle \epsilon^3 \rangle = t^3 N_3$). We can thus spot one of the most prominent