mechanisms for frustration (which determines, e.g., the magnetic properties of fcc lattices) by just looking at the DOS and measuring its third moment.[17] More generally, a symmetric DOS implies irrelevance or cancellation of loops with odd length.

Application to the Bethe lattice

An application of the above formalism to the Bethe lattice is particularly simple, since the only self-avoiding loops have length 2 and thus the series in (2.29) terminates at the first term,

$$G^{-1}(\omega) = \omega - G(\omega) \tag{2.34}$$

from which (2.25) follows directly. Introduction of NNN hopping on the Bethe lattice completely changes its topology and introduces loops of arbitrary order as is illustrated in Fig. 2.11. For $d = \infty$ and unit variance, we have in this case

$$a_3 = t'^*(t^{*2} + t'^{*2}); \quad a_4 = 8t^{*2}t'^{*2} + t'^{*4}; \quad a_5 = 20t^{*2}t'^{*3} + t'^{*5}\,. \tag{2.35}$$

Already at this point we can see that the third moment is nonvanishing for all finite t'^* and consequently the DOS must be asymmetric. This result contradicts a claim of the Kotliar group (Rozenberg, Kotliar, Kajüter, Thomas, Rapkine, Honig, and Metcalf, 1995) that the DOS of the Bethe lattice with NN and NNN hopping should be always semi-elliptic.[18] While it might be possible to find a closed solution for a_n for arbitrary n and thus compute reliable numerical estimates via Padé approximants, it is clearly impossible to obtain a closed solution for G and the DOS in the general case along this route (it would, in fact, be very surprising when the full exact result could be written in closed form at all). At least, we are able to present the full solution for the Bethe lattice with NNN, but no NN hopping (i.e., for $t^* = 0$). For this special case and unit variance we have $a_n = 1$ for all n due to the very special topology in this limit where all sites visited by each self-avoiding loop of length $n \geq 3$ are NN of the same site (not on the loop). Therefore (2.29) reduces to a geometric series and we obtain for $Z = \infty$[19]

$$G(\omega) = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{1}{1+\omega}} \tag{2.36}$$

$$\rho(\omega) = \frac{1}{2\pi}\frac{\sqrt{3-\omega}}{\sqrt{1+\omega}}, \tag{2.37}$$

which agrees with the model DOS (2.27) for the case of extreme asymmetry ($a = 1$) when the frequency is shifted by $\omega \to \omega+1$. Thus, we have shown that the model DOS

[17] This observation implies that 3-loops do not survive the limit $d \to \infty$ for the hc lattice with extended hopping along the axes (Müller-Hartmann, 1989*a*) which we verified in subsection 2.1.3 and makes the mechanism for magnetic frustration present in this lattice type appear highly pathological.

[18] In fact, our observation essentially invalidates all results concerning the microscopic treatment of frustration in the cited paper. We mention in passing that the sum rule for the optical conductivity stated in this paper is also incorrect, even for the Bethe lattice with only NN hopping (cf. Sec. 4.4).

[19] The generalization to finite Z seems straightforward and might be considered in an upcoming publication.