interpolates between Bethe lattices for $Z = \infty$ with only NN and only NNN hopping. In order to exclude that there is an accidental connection to the microscopic model also for $0 < a < 1$, we determined the leading nontrivial corresponding moments of the DOS (2.27) for $a = 0.9$ (which are $\langle(\epsilon - \langle\epsilon\rangle)^3\rangle \approx 0.6268$, $\langle(\epsilon - \langle\epsilon\rangle)^4\rangle \approx 2.3929$, $\langle(\epsilon - \langle\epsilon\rangle)^5\rangle \approx 3.3802$) and fixed the ratio $t'^*/t^* \approx 0.2208$ to fulfill (2.32) and (2.35). This leads to $a_4 = 0.3568$, $a_5 = 0.1917$ and corresponds to 4th and 5th moments of 2.3568 and 4.0720, respectively. Clearly, the results are not compatible; a_5 would even have to have the opposite sign in order to reproduce the 5th moment of the model DOS.

From the above we may further conclude that the weights a_n for self-avoiding loops of length $n > 2$ must cancel exactly also for the tight-binding Hamiltonian having a semi-elliptic DOS which we will construct in the following section. While this consideration implies that the Bethe tree is the only (pseudo)lattice with semi-elliptic DOS and only one nonzero hopping parameter it could in principle also be used to verify or even construct Hamiltonians. In practice, however, there exist many contributions, e.g., to the sum over 4-loops on hypercubic lattices, even for a small set of longer range hopping terms. For hopping t to NN and t_3 to 3rd-nearest neighbors we found on the hc lattice the leading contribution

$$a_4 \approx t^{*4} + 36t^{*2}t_3^{*2} + 4\sqrt{6}t^{*3}t_3^* + 48\sqrt{6}t^*t_3^{*3} + 108t_3^{*4} \tag{2.38}$$

which adds up to -0.04 when the values from Table 2.1 (page 55) are used. Contributions from longer-range hopping are larger than might be expected from the small value of this partial sum. One contribution is, e.g., $12\sqrt{20}t^{*2}t_3^*t_5^* \approx -0.31$. We find it remarkable that the semi-elliptic DOS imposes such severe constraints (i.e., the cancellation of all loops of length greater than 2) on corresponding tight-binding models, even though this criterion appears not to be of general constructive value.

Off-diagonal disorder

One further application where the RPE can provide much insight is off-diagonal disorder. For independent random variables t_{ij}, all contributions from self-avoiding loops of order $n > 2$ vanish exactly as long as the hopping elements vanish on average, $\langle t_{ij}\rangle = 0$. The remarkable conclusion is that noninteracting Green function and DOS agree with that of the Bethe lattice (with NN hopping) for arbitrary topology in this case. This statement, valid for $Z = \infty$, is more general than Wegner's treatment of "gauge invariant" tight-binding models which predicts local Bethe lattice properties only for systems with a fully symmetric distribution of hopping matrix elements, $\langle t_{ij}^{2n+1}\rangle = 0$ (Wegner, 1976). In our view, the tighter restrictions for finite Z can be linked to the fact that in this case not only the weight of self-avoiding loops, but of all loops (of length $n > 2$) has to vanish in order to reduce the effective mean free path to one lattice spacing, which is the physics of all models with Bethe DOS. In the context of DMFT calculations, frustration caused by disorder of the above type was considered for a fully connected model (Georges and Krauth, 1993) and then also for the more general cases (Dobrosavljević and Kotliar, 1993; Dobrosavljević and Kotliar, 1994).