As a last conclusion from this section let us note that additional hopping (e.g., along NNN bonds) does *not* modify the semi-elliptic DOS of the Bethe lattice when these additional bonds are fully disordered (with zero mean hopping amplitudes). In contrast to the clean case it is, thus, possible to tune magnetic frustration in such a system by variation of $\langle t_2^{*2} \rangle$ at constant DOS.[20]

2.3 General Density of States in $d = \infty$

In this section, we will present a general algorithm for finding an isotropic, translationally invariant tight-binding Hamiltonian corresponding to an arbitrary DOS in $d = \infty$. While the availability of such a method to construct a microscopic Hamiltonian corresponding to some model DOS might be useful in any dimension, it is absolutely essential for considering spatially inhomogeneous phases (e.g, antiferromagnetism or charge density waves) or computing transport properties within the DMFT. The algorithm developed in this section will later enable us to perform the first fully consistent DMFT calculation of the optical conductivity for a periodic and isotropic lattice (cf. footnote 3 on page 28) with the Bethe semi-elliptic DOS (see Sec. 4.4).

Scaling Relations

We start by rewriting the usual noninteracting Hamiltonian with general (translationally invariant) hopping,[21]

$$\hat{H}_0 = \sum_{ij,\sigma} t_{ij}\, \hat{c}^\dagger_{\boldsymbol{R}_i\sigma} \hat{c}_{\boldsymbol{R}_j\sigma} = \sum_{i,\sigma} \sum_{\boldsymbol{\tau}} t_{\boldsymbol{\tau}}\, \hat{c}^\dagger_{\boldsymbol{R}_i\sigma} \hat{c}_{\boldsymbol{R}_i+\boldsymbol{\tau},\sigma} \tag{2.39}$$

$$= \sum_{\boldsymbol{k},\sigma} \epsilon(\boldsymbol{k}) \hat{n}_{\boldsymbol{k}\sigma}, \tag{2.40}$$

where

$$\epsilon(\boldsymbol{k}) = \sum_{\boldsymbol{\tau}} t_{\boldsymbol{\tau}} \mathrm{e}^{i\boldsymbol{\tau}\cdot\boldsymbol{k}} = \sum_{\boldsymbol{\tau}} \left(\frac{t_{\boldsymbol{\tau}} + t^*_{\boldsymbol{\tau}}}{2} \cos(\boldsymbol{\tau}\cdot\boldsymbol{k}) + i \frac{t_{\boldsymbol{\tau}} - t^*_{\boldsymbol{\tau}}}{2} \sin(\boldsymbol{\tau}\cdot\boldsymbol{k}) \right). \tag{2.41}$$

Classifying the hopping vectors $\boldsymbol{\tau}$ according to their length measured by the taxi-cab metric, we may split up the kinetic energy accordingly,

$$\epsilon(\boldsymbol{k}) = \sum_{D=1}^{\infty} \epsilon_D(\boldsymbol{k}), \qquad \epsilon_D(\boldsymbol{k}) = \sum_{\|\boldsymbol{\tau}\|=D} t_{\boldsymbol{\tau}} \mathrm{e}^{i\boldsymbol{\tau}\cdot\boldsymbol{k}}. \tag{2.42}$$

For $D > 1$, each set of vectors $\{\boldsymbol{\tau}; ||\boldsymbol{\tau}|| = D\}$ consists of different subsets which are pairwise inequivalent such as, for $D = 2$, the subsets $\{\boldsymbol{\tau}; \boldsymbol{\tau} = \pm \boldsymbol{e}_\alpha \pm \boldsymbol{e}_\beta; \alpha \neq$

[20] The same applies to the regular, translation invariant model with semi-elliptic DOS which we will construct in the next section: additional even-range disorder hopping terms could introduce frustration without changing the DOS.

[21] For sake of clarity, we write $\epsilon(\boldsymbol{k})$ instead of $\epsilon_{\boldsymbol{k}}$ for the full dispersion [i.e., the full sum in (2.42)] throughout this section.