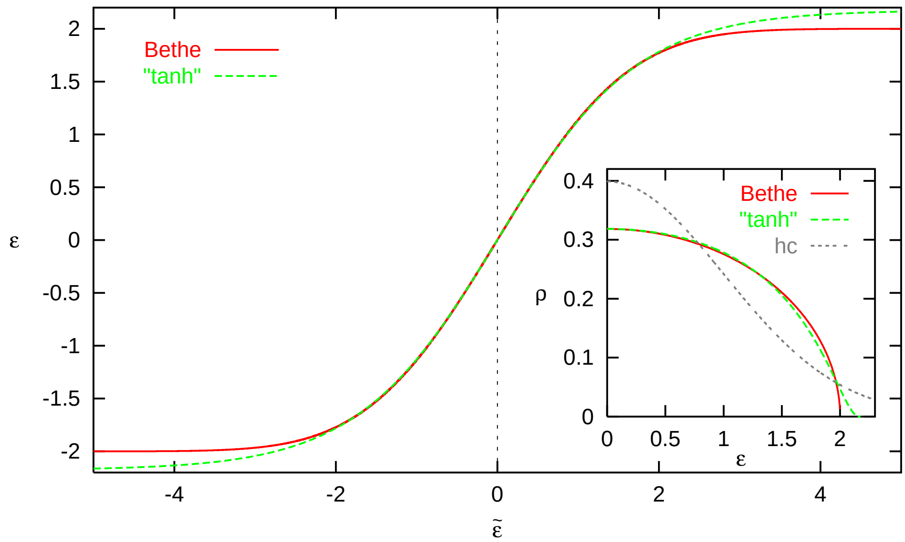

Figure 2.16: Transformation function $\epsilon = \mathcal{F}(\tilde{\epsilon})$ for the semi-elliptic Bethe DOS: numerically exact result and approximation by a hyperbolic tangent according to (2.69). The inset shows corresponding DOSs [cf. (2.57)] in comparison to the Gaussian DOS of the hc lattice.

D	t^*_D	$\sum_{n=1}^D {t^*_n}^2$	D	t^*_D	$\sum_{n=1}^D {t^*_n}^2$
1	0.98731	0.974773	9	0.00343	0.999999
3	-0.15353	0.998345	11	-0.00108	1.000000
5	0.03893	0.999861	13	0.00033	1.000000
7	-0.01125	0.999987	15	-0.00011	1.000000

Table 2.1: *Leading scaled hopping matrix elements corresponding to a semi-elliptic Bethe DOS. The columns consisting of partial sums indicate rapid convergence of the series: only a fraction* $1 - \sum_{n=1}^D {t^*_n}^2$ *of the variance arises from hopping terms beyond* D^{th} *neighbors. The mean squared hopping distance is* $\sum_{D=1}^\infty D\, {t^*_D}^2 \approx 1.05406$.

only a fraction 10^{-6} results from hopping beyond 9^{th}-nearest neighbors. Since such a short average range is certainly within physical limits for any itinerant system, our model seems a no less natural description for systems involving narrow tight-binding orbitals than the Hubbard model (with only NN hopping) itself. The situation would be different if we had found slow algebraic decay of the hopping amplitudes like we have seen for the toy flat band example, (2.67), which made the latter appear rather unphysical.

Using (2.62) and (2.63), we can compute the averaged squared Fermi velocity $\langle|\boldsymbol{v_k}|^2\rangle(\epsilon)$, where for each value of ϵ the average is taken over all momenta $\boldsymbol{k}$ with