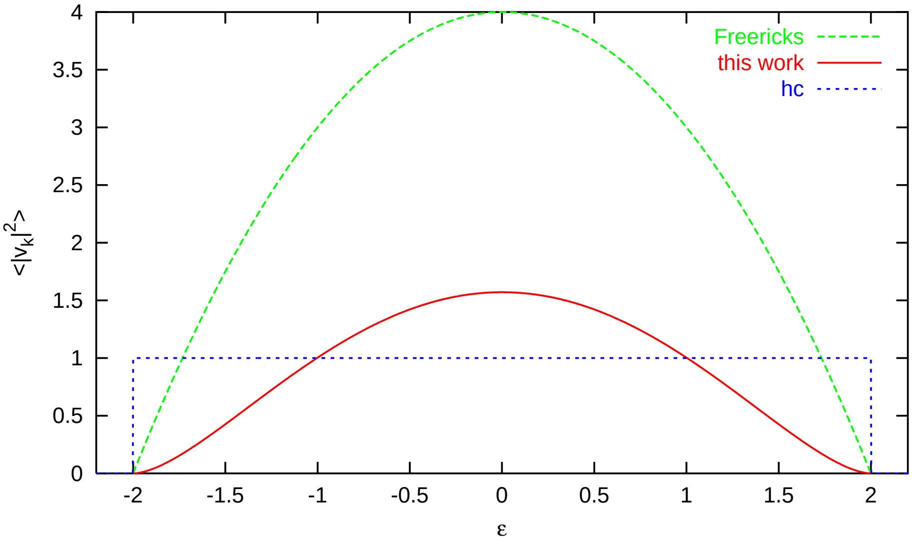

Figure 2.17: Averaged squared Fermi velocity of the isotropic lattice model with Bethe DOS (solid line). As required physically, it vanishes at the band edges. This has to be contrasted with the constant value used in calculations so far which is only exact for the hc lattice. Also shown is the parabolic form proposed by Chung and Freericks (1998*a*).

$\epsilon_{\boldsymbol{k}} = \epsilon$ in closed form for unit lattice spacing $a = 1$:[27]

$$\langle|\boldsymbol{v_k}|^2\rangle(\epsilon) = \frac{\pi}{2(1-\epsilon^2/4)} \exp\left[-2\left(\mathrm{erf}^{-1}\left(\frac{\epsilon\sqrt{1-\epsilon^2/4}+2\sin^{-1}(\epsilon/2)}{\pi}\right)\right)^2\right] \tag{2.70}$$

Here, we have used the fact that $\langle|\boldsymbol{v_k}|^2\rangle(\epsilon)$ is effectively constant (and equals 1 for unit variance and lattice spacing) in the hypercubic case. The result, shown in Fig. 2.17, has all the qualitative features expected for this observable in any dimension: $\langle|\boldsymbol{v_k}|^2\rangle(\epsilon)$ is maximal near the band center, strongly reduced for large (absolute) energies and vanishes at the band edges: states at the (noninteracting) band edge do not contribute to transport. For the hc lattice, the band edges are at $\pm\infty$, so that a constant $\langle|\boldsymbol{v_k}|^2\rangle(\epsilon)$ is somewhat pathological, but at least consistent. This consistency is lost when a formalism derived for the hc lattice is applied to the Bethe DOS. While a more complete discussion of this point has to be postponed to chapter 4, we note that this problem (which had otherwise been ignored in the literature) led Freericks to propose the parabolic form[28] also shown in Fig. 2.17. The resulting computation of the optical conductivity was the first one to go beyond the hc lattice

[27] More formally, we may use equations (2.1) and (2.2) to define $\langle|\boldsymbol{v_k}|^2\rangle(\epsilon) := \tilde{\rho}(\epsilon)/\rho(\epsilon)$.

[28] One may ask if one can find any model for which $|\boldsymbol{v_k}|^2$ shows this behavior. Using our formalism, namely (2.63), we obtain a differential equation for the energy transformation function, $|\mathcal{F}'(\tilde{\epsilon})| = \sqrt{4-\mathcal{F}^2(\tilde{\epsilon})}$, which is solved by $\mathcal{F}(\tilde{\epsilon}) = 2\sin(\tilde{\epsilon}+\tilde{\epsilon}_0)$. This is by no means a monotonic function, however, which violates the assumption under which it was derived. Therefore, we conclude that the behavior assumed by Freericks cannot occur for the class of tight-binding Hamiltonians considered within our method.