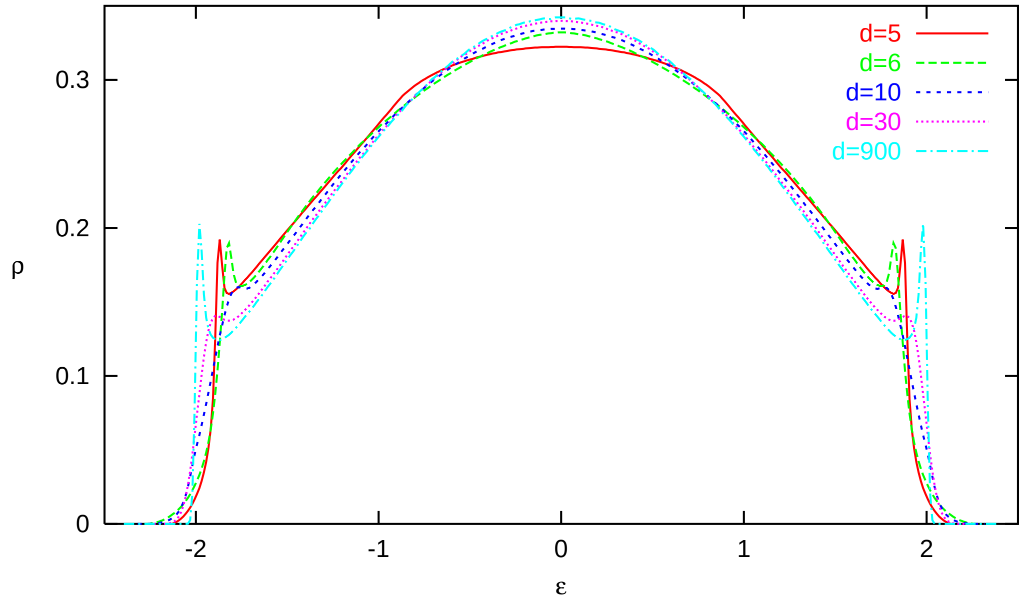

Figure 2.20: DOS of the isotropic tight-binding model with semi-elliptic DOS, truncated at 3$^{\text{rd}}$ order and applied in finite dimensions.

$$
\begin{aligned}
A_3(\boldsymbol{k}) &= \Big(\sum_\alpha \cos(k_\alpha)\Big)^3 - 3\sum_\alpha \cos^2(k_\alpha)\sum_\alpha \cos(k_\alpha) + \sum_\alpha \cos^3(k_\alpha) \\
A_4(\boldsymbol{k}) &= \Big(\sum_\alpha \cos(k_\alpha)\Big)^4 - 6\Big(\sum_\alpha \cos(k_\alpha)\Big)^2 \sum_\alpha \cos^2(k_\alpha) + 3\Big(\sum_\alpha \cos^2(k_\alpha)\Big)^2 \\
&\quad +8\sum_\alpha \cos^3(k_\alpha)\sum_\alpha \cos(k_\alpha) - 6\sum_\alpha \cos^4(k_\alpha) \\
A_5(\boldsymbol{k}) &= \Big(\sum_\alpha \cos(k_\alpha)\Big)^5 - 20\sum_\alpha \cos^2(k_\alpha)\sum_\alpha \cos^3(k_\alpha) - 30\sum_\alpha \cos(k_\alpha)\sum_\alpha \cos^4(k_\alpha) \\
&\quad +20\Big(\sum_\alpha \cos(k_\alpha)\Big)^2 \sum_\alpha \cos^3(k_\alpha) + 15\sum_\alpha \cos(k_\alpha)\Big(\sum_\alpha \cos^2(k_\alpha)\Big)^2 \\
&\quad -10\Big(\sum_\alpha \cos(k_\alpha)\Big)^3 \sum_\alpha \cos^2(k_\alpha) + 24\sum_\alpha \cos^5(k_\alpha)\,.
\end{aligned}
$$

The resulting DOSs for various dimensions and a hopping range truncated at $D_{\max} = 3$ have been evaluated by Monte Carlo sampling of the momentum integrals and are shown in Fig. 2.20. As expected, the singularities in the DOS observed for $d = \infty$ are smoothed out in finite dimensions, so that the agreement with a Bethe DOS is even improved. The value at $\epsilon = 0$ is slightly too high (which is accidentally offset by a finite-d error for $d = 5$). For even lower dimensions $d = 3$ and $d = 4$ (not shown, cf. subsection 4.4.5), the shape of densities of states is dominated by Van-Hove singularities rather than the influence of 3$^{\text{rd}}$-nearest–neighbor hopping.

Both the overall deviations from the Bethe DOS and the dimensional dependence