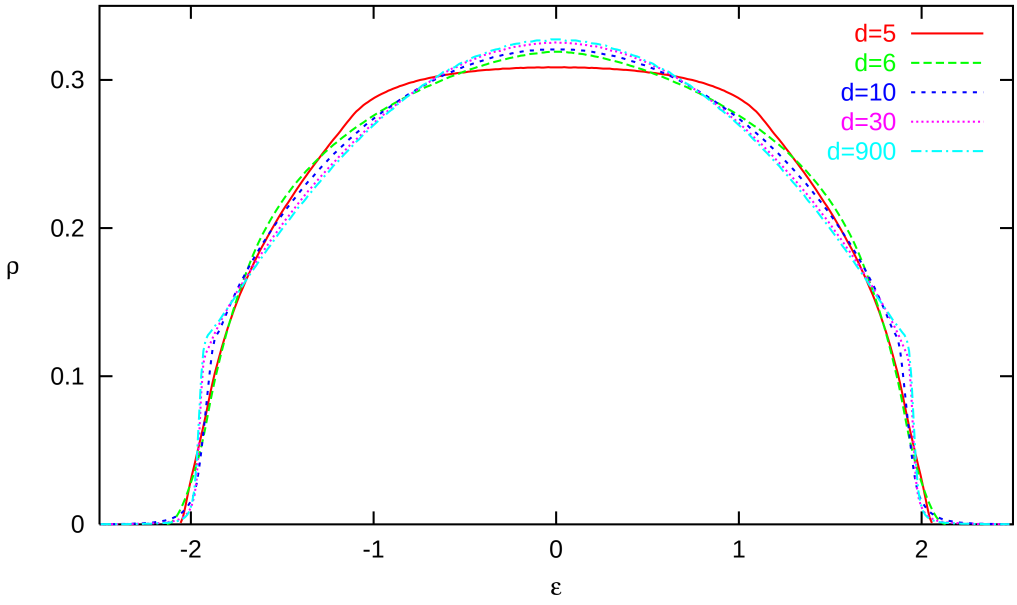

Figure 2.21: DOS of the isotropic tight-binding model with semi-elliptic DOS, truncated at 5^{th} order and applied in finite dimensions. Accidentally, the curve for $d = 10$ best approximates the semi-elliptic Bethe DOS.

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are much smaller when the hopping range is extended to 5^{th}-nearest neighbors as shown in Fig. 2.21. It is remarkable that a reasonable approximation to the Bethe DOS is obtained even for $d = 5$, although the underlying tight-binding model was derived for $d \gg D_{\max}$. While the dimensional dependence is expected to further decrease significantly when the hopping range is extended to $D_{\max} \geq 9$, we did not find it worth the effort to extend these finite-dimensional studies beyond $D_{\max} = 5$ after having successfully demonstrated the overall robustness of the method. The weak dimensional dependence seen in Fig. 2.21 should be contrasted with the very strong dependence of the DOS on dimensionality observed for other lattice types, e.g., for the fcc lattice shown in Fig. 2.7.

One useful feature of our method is that (for a monotonic transformation function) the energy minimum remains associated with $\boldsymbol{k} \approx \boldsymbol{0}$ and the energy maximum associated with momenta $\boldsymbol{k} \approx \boldsymbol{Q}$. This can be illustrated in finite dimensions, where these vectors (which are nongeneric[31] in the limit $Z \to \infty$) still have finite weight. In Fig. 2.22 a and Fig. 2.22 b we show the full DOS of the hc lattice and of our model truncated at 5^{th} order for dimension $d = 10$, respectively (dotted lines). The solid and dashed lines in the same figures indicate partial DOSs which were obtained by taking the sum in (2.1) either only for momenta with $|\boldsymbol{k}| < 2d$, $|\boldsymbol{k} - \boldsymbol{Q}| < 2d$, or with $|\boldsymbol{k}|/\langle|\boldsymbol{k}|\rangle < 0.15$. Here, the average length of a momentum vector is approximated by its asymptotic for $Z \to \infty$, $\langle|\boldsymbol{k}|\rangle \approx \pi\sqrt{d/3}$ (cf. App. A.2). Evidently, generic states of average length are least affected by the transformation. Therefore, our model may

[31] For a discussion of generic versus nongeneric momenta, see App. A.2.