transport properties pointless. Due to the violation of symmetry between $\boldsymbol{k} = \boldsymbol{0}$ and $\boldsymbol{k} = \boldsymbol{Q}$, concepts like perfect nesting become meaningless. Additionally, the hopping is dominantly along the axes and is very slowly decaying (as $t_D \sim D^{-2}$, only slightly faster than in the one-dimensional case). Thus, our approach appears much more physical in higher dimensions which is not surprising since it was derived for the large-d limit.[32]

2.5 Conclusion

In this chapter, we have carefully studied the relationships between lattice types, densities of states, and magnetic frustration in high dimensions. Evaluating Brillouin-zone integrals by Monte Carlo sampling, we have computed the (noninteracting) DOS of the hypercubic lattice with hopping to nearest and/or next-nearest neighbors as well as the DOS of the hyperdiamond lattice (see App. B) in low, intermediate, and high dimensions. Thereby and by comparing with the corresponding analytical $d = \infty$ results we have distinguished generic features from features that are artifacts of the $d = \infty$ limit. In particular, we have established that for regular lattices, magnetic frustration is generically associated with an asymmetric DOS or with off-diagonal disorder.

We have then characterized the Bethe "lattice" which is a tree in the sense of graph theory rather than a regular lattice. We have pointed out that a semi-elliptic DOS implies the absence or cancellation of all self-avoiding loops in a renormalized perturbation expansion for any lattice. By computing moments of the DOS of the Bethe lattice with hopping to nearest and next-nearest neighbors (NNN), we have contradicted an earlier claim that NNN hopping should leave the DOS invariant. Furthermore, we have presented the first (analytic) calculation of the (strongly asymmetric) DOS of the Bethe lattice with only NNN hopping.

We have developed a general method of constructing a tight-binding Hamiltonian which yields a given, arbitrary DOS in the limit of infinite dimensions. This method allows for the computation of all physical quantities that can be defined for the corresponding finite-dimensional model. Such a method is particularly useful in the context of the DMFT, where nonlocal properties can be derived by combining an essentially local property of the interacting system with a nonlocal property of the noninteracting system. Examples for such properties are the optical conductivity (which we will consider in chapter 4) and susceptibilities.

By applying the new general-dispersion formalism to the semi-elliptic "Bethe" DOS, we have found the first regular lattice with a bounded DOS in high dimensions, thereby removing one of the major artifacts previously regarded as inherent to the limit $d \to \infty$. We have verified that versions of this model with truncated hopping ranges still give reasonable descriptions. While the DOS is then no longer strictly bounded, the weight associated with long-frequency tails is suppressed exponentially with the number of hopping terms and becomes negligible for numerical purposes

[32] In contrast, only Kollar's construction can be applied in very low dimensions, in particular in one dimension where it becomes exact.