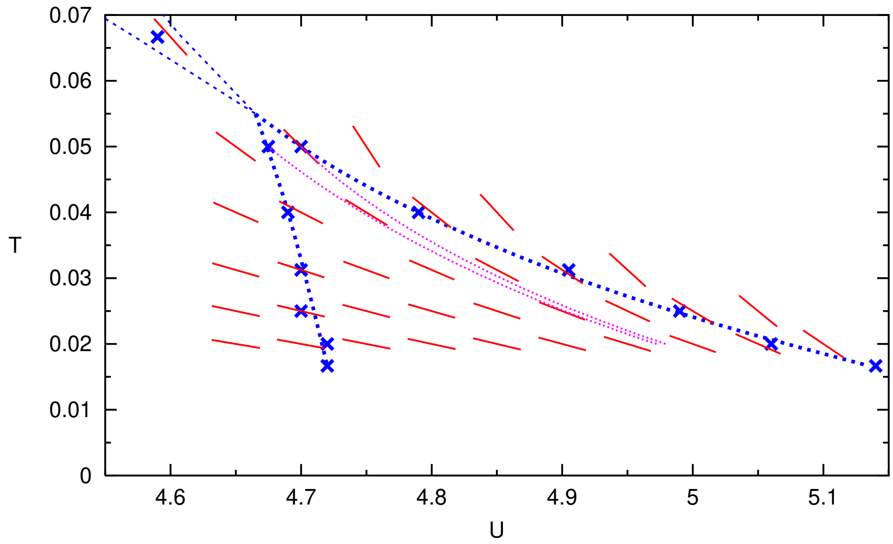

Figure 3.44: Gradient field for $U_c(T)$ as implied by the differential equation (3.30), symbolized by short solid line segments. The symbols and short-dashed lines represent the boundaries of coexistence region and crossover region (cf. Fig. 3.31). The dotted lines show solutions of the (linearized) differential equation with the initial condition $U_c = U_{c1}$ and $U_c = U_{c2}$ for $T = 0.05$, respectively, which bracket the final solution.

Figure 3.44 illustrates that the procedure of integrating the solution towards lower temperatures, starting at the tip of the coexistence region is inherently stable. Here, the crosses and short-dashed curves indicate the coexistence and crossover regions determined in Fig. 3.31. The slopes of each short solid line with center U_i, T_i is given by the value $f(T_i, U_i)$ which can be read off from Fig. 3.43 a. Since the slope of any solution of the differential equation (3.30) has to agree with $f(T, U)$ at each point, solutions practically cannot cross the slopes shown in Fig. 3.44 (within their error bars and in the limit of vanishing lengths). The fast increase of the absolute slope from left to right makes them act like a funnel for the integration of solutions towards lower T. Therefore, the impact of errors in the position of the tip of the coexistence region (and also of $f(T, U)$ at high T) on the solutions decays rapidly with decreasing T. This is illustrated by the two test solutions (dotted lines) shown in Fig. 3.44 which fulfill the initial conditions $U(0.05) = U_{c1}(0.05)$ and $U(0.05) = U_{c2}(0.05)$, respectively. Although these lines, which bracket the physical solution $U_c(T)$, are far apart initially, their distance is already very small for $T = 0.02$.

3.6.2 Low-temperature Asymptotics of $U_c(T)$

While the data presented so far suffices for an accurate determination of the thermodynamic phase boundary $U_c(T)$ within the range of temperatures $T \geq 0.02$ covered by QMC simulations, the quality of an extrapolation to even lower temperatures based on the QMC data for $f(T, U)$ alone would be questionable. By including additional low-temperature information, however, we will be able to reliably integrate the solu-