which can be Fourier-transformed to yield

$$J_\alpha(\boldsymbol{q},\omega) = \sum_\beta \sigma_{\alpha\beta}(\boldsymbol{q},\omega) E_\beta(\boldsymbol{q},\omega). \tag{4.4}$$

Since photons have a much steeper (linear) energy dispersion than electrons within the Brillouin zone, the transmitted momentum is essentially zero for not too high energies. Thus, in the optical frequency regime and below we can take the long-wavelength limit $\boldsymbol{q} \to 0$. For cubic symmetry, the response is then isotropic, i.e., the conductivity tensor becomes diagonal:[3]

$$\boldsymbol{J}(\omega) = \sigma(\omega)\boldsymbol{E}(\omega). \tag{4.5}$$

Note that causality, i.e., the restriction to $t' < t$ in (4.2) and (4.3), implies a retarded conductivity $\sigma(\omega)$ which is analytic in the upper half plane.

4.1.1 Connection between Conductivity and Reflectivity

Under the conditions used in the derivation of (4.4), the dielectric function $\epsilon(\omega)$ is defined as relating the (coarse-grained) electrical displacement $\boldsymbol{D}$ to the (coarse-grained) electric field $\boldsymbol{E}$,[4]

$$\boldsymbol{D}(\omega) = \epsilon(\omega)\boldsymbol{E}(\omega). \tag{4.6}$$

The dielectric function can be directly expressed in terms of the optical conductivity,

$$\epsilon(\omega) = 1 + \frac{4\pi i \sigma(\omega)}{\omega}. \tag{4.7}$$

Note that here $\sigma(\omega)$ here has the dimension of a frequency (or energy) which is only valid in the Gaussian system of units. Also note the importance of the absolute scale of $\sigma(\omega)$ which determines qualitative features of $\epsilon(\omega)$ as, e.g., the zeros of $\epsilon_1(\omega)$. For later use, we also state the equivalent equation valid in the SI system [$\epsilon \to \epsilon/\epsilon_0$, $\sigma \to \sigma/(4\pi\epsilon_0)$]

$$\epsilon_{\mathrm{SI}}(\omega) = \epsilon_0 + \frac{i\sigma(\omega)}{\omega}; \quad \epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{Fm}^{-1} = 8.85 \times 10^{-12}\,\frac{\mathrm{s}}{\Omega\mathrm{m}}. \tag{4.8}$$

In some cases, solids can be prepared in thin enough slices that absorption and phase shift can be directly measured in transmission experiments in the energy ranges of interest. Then, the index of diffraction $n(\omega)$ determines the phase velocity $c_{\mathrm{medium}}(\omega) = c/n(\omega)$, i.e., the phase shift in a sample of finite thickness, while $k(\omega)$ quantifies the absorption per wavelength [see, e.g., Hecht (1997) or Landau and Lifschitz (1990)]. The functions $\epsilon(\omega)$ and $\sigma(\omega)$ are obtained from such measurements via

$$\kappa(\omega) \equiv \sqrt{\epsilon(\omega)} = n(\omega) + ik(\omega)\,, \tag{4.9}$$

[3]This is not true at finite $\boldsymbol{q}$, even in an homogeneous system. Later, in Sec. 4.4, we will also need to consider the more general case where $\sigma(\omega)$ is still diagonal, but not isotropic.

[4]Unless stated otherwise, we will here and in the following use the Gaussian system of units (cgs).