where the square root is taken such that n is positive, and via (4.7).

If transmission experiments are not possible, but at least good (i.e., smooth and clean) surfaces can be prepared, then optical properties can still be probed in reflection mode. The reflectivity for normally incident light is

$$r(\omega) = \left|\frac{1-\kappa(\omega)}{1+\kappa(\omega)}\right|^2 = \frac{\big(1-n(\omega)\big)^2 + k^2(\omega)}{\big(1+n(\omega)\big)^2 + k^2(\omega)}, \tag{4.10}$$

Since $\kappa(\omega)$ [like $\sigma(\omega)$ and $\epsilon(\omega)$] is analytic in the upper half plane,[5] its real part $n(\omega)$ and its imaginary part $k(\omega)$ are related by Kramers-Kronig transformations. Using information about $r(\omega)$ at all frequencies, one can extract n and k by Kramers-Kronig analysis which yields $\epsilon(\omega)$ and finally $\sigma(\omega)$. This procedure and the implicit extrapolations to frequencies outside the range of measurements can be avoided by measurements at variable angle Θ and/or polarization of the incident light. In this case, the reflectivity is given (for permeability $\mu = 0$, i.e., nonmagnetic materials) by (Stratton, 1941)

$$r_\perp(\Theta,\omega) = \frac{\big(q(\Theta,\omega)-\cos\Theta\big)^2 + p^2(\Theta,\omega)}{\big(q(\Theta,\omega)+\cos\Theta\big)^2 + p^2(\Theta,\omega)}, \tag{4.11}$$

$$r_\parallel(\Theta,\omega) = r_\perp(\Theta,\omega)\frac{\big(q(\Theta,\omega)-\sin\Theta\tan\Theta\big)^2 + p^2(\Theta,\omega)}{\big(q(\Theta,\omega)+\sin\Theta\tan\Theta\big)^2 + p^2(\Theta,\omega)}, \tag{4.12}$$

where all quantities are real and [$n \equiv n(\omega)$, $k \equiv k(\omega)$ as above]

$$p^2(\omega) = \frac{1}{2}\left[k^2 - n^2 + \sin^2\Theta + \sqrt{4n^2k^2 + (n^2-k^2+\sin^2\Theta)^2}\right], \tag{4.13}$$

$$q^2(\omega) = \frac{1}{2}\left[n^2 - k^2 - \sin^2\Theta + \sqrt{4n^2k^2 + (n^2-k^2+\sin^2\Theta)^2}\right]. \tag{4.14}$$

Since p^2 and q^2 reduce to k^2 and n^2, respectively, for normal incidence ($\Theta = 0$) it is easily checked that (4.11) and (4.12) indeed reduce to (4.10) in this limit.

Transmission experiments, if possible, are not only preferable over reflection experiments since the Kramers-Kronig analysis or mathematical complications are avoided, but also, since it is more certain that bulk properties are measured (as opposed to surface properties).

For illustration, the dielectric functions of a single-mode Lorentz oscillator model[6] are shown in Fig. 4.1. Here, we have chosen a nonzero oscillator frequency with a comparatively small damping term so that the model describes an insulator. As we

[5] At finite $\boldsymbol{q}$, this is rigorously true only for the transverse part $\epsilon_t(\boldsymbol{q},\omega)$; also analytic is the inverse of the longitudinal part, $\epsilon_l^{-1}(\boldsymbol{q},\omega)$ (v. Baltz, 1997). This distinction vanishes, however, for $\boldsymbol{q} = 0$.

[6] This model is based on damped harmonic oscillator modes with weight f_j, resonance frequency ω_j, and damping term γ_j, leading to $\epsilon(\omega) = 1+4\pi e^2/m\sum_j f_j/((w_j^2-w^2)-2i\gamma_j w)$; see, e.g., Altarelli, Dexter, Nussenzveig, and Smith (1972). We use it here as a convenient fit formula producing an analytic function with a strongly peaked imaginary part. On a phenomenological level, more realistic curves are easily obtained by using several oscillator levels.