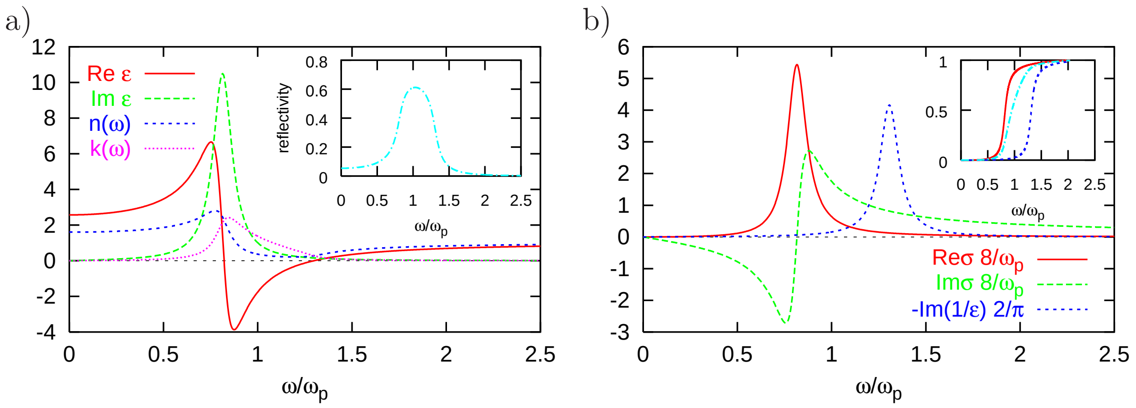


**Figure 4.1:** Scheme: dielectric functions in insulator (Lorentz oscillator model). **a)** dielectric function $\epsilon(\omega)$, refractive index $n(\omega)$, and absorption coefficient $k(\omega)$. Inset: reflectivity for normal incidence. **b)** Scaled optical conductivity $\sigma(\omega)$ and electronic energy loss function $\operatorname{Im}\frac{-1}{\epsilon(\omega)}$. Inset: integrals $\int\frac{\mathrm{d}\omega}{\omega_\mathrm{p}}\frac{8\sigma(\omega)}{\omega_\mathrm{p}}$ (solid line), $\int\frac{\mathrm{d}\omega}{\omega_\mathrm{p}}\frac{\omega}{\omega_\mathrm{p}}\frac{4}{\pi}k(\omega)$ (dash-dotted line), and $\int\frac{\mathrm{d}\omega}{\omega_\mathrm{p}}\frac{\omega}{\omega_\mathrm{p}}\frac{2}{\pi}\operatorname{Im}\frac{-1}{\epsilon(\omega)}$ (dotted line) related to the sum rules (4.18)-(4.20).

will discuss in the following subsection 4.1.2, $\epsilon(\omega) \overset{\omega\to\infty}{\longrightarrow} 1-\omega_\mathrm{p}^2/\omega^2$ for some plasma frequency $\omega_\mathrm{p}$ which we used here to rescale $\omega$ and $\sigma(\omega)$ so that the resulting plots are dimensionless. Near the oscillator frequency $\omega_0 \approx 0.82\,\omega_\mathrm{p}$, the rapid changes in $\operatorname{Re}\epsilon(\omega)$ and $n(\omega)$ with negative slope show anomalous dispersion. Here, the usual expression $u = c/(1+\mathrm{d}n/\mathrm{d}\omega)$ for the group velocity of light would imply $u > c$. It is not applicable in this frequency range, however, due to the strong absorption indicated by peaks in $\operatorname{Im}\epsilon(\omega)$, $\operatorname{Re}\sigma(\omega)$, and $k(\omega)$. The peak of the reflectivity is at slightly higher frequencies $\omega \approx \omega_\mathrm{p}$ (inset of Fig. 4.1 a). In our example, the absorption is so strong that $\operatorname{Re}\epsilon(\omega)$ changes sign at intermediate frequencies. When it vanishes outside the absorption region (here at $\omega_{p,\mathrm{bulk}} \approx 1.3\,\omega_\mathrm{p}$), the system is susceptible to a bulk plasma oscillation since (4.6) then allows for finite electric field without external field, $D \equiv 0$. This oscillator mode can be excited by injecting high-energy electrons which then lose energy in quanta of $\hbar\omega_{p,\mathrm{bulk}}$. More precisely, the distribution of energy losses for single scattering (transmission through thin slices) is approximately proportional to the electron energy-loss function $-\operatorname{Im}\{1/\epsilon(\omega)\}$ depicted in Fig. 4.1 b.

### 4.1.2 Optical $f$-sum Rules

The so-called $f$-sum rules are based on the Thomas-Reiche-Kuhn oscillator sum rule (Thomas, 1925; Kuhn, 1925; Reiche and Thomas, 1925) for atomic spectra which was later generalized to the momentum-dependent case (Bethe, 1930). On a phenomenological level, the sum rules follow from analytic properties of the dielectric functions and the very general assumption that absorptive processes only extend over a finite frequency range. Their common unknown parameter, i.e., the plasma frequency $\omega_\mathrm{p}$, can only be determined on a microscopic level.