value of the plasma frequency only depends on the total electron density n (for an electron charge e and mass m),

$$\omega_{\mathrm{p}}^2 = \frac{4\pi n e^2}{m}\,, \tag{4.21}$$

and additional exact sum rules may be derived (Nozières and Pines, 1958; Nozières and Pines, 1959; Nozières and Pines, 1989). It is an interesting question to what extent the formalism and this simple result carry over to real systems. The full sum rules remain valid for real solids (Nozières and Pines, 1989); for a theoretical analysis, see subsection 4.2.1. At least for light metals like lithium or beryllium (where relativistic effects are negligible and the homogeneity condition can be met) the integrals (4.18)-(4.20) should, thus, be observable with ω_{p} given by (4.21). Experimental data at soft X-ray frequencies $\hbar\omega \gtrsim 100$ eV is, however, scarce. Only after an extensive literature search we could find an accurate experimental verification of the full sum rule for aluminum (see next subsection).

Alternatively, one may consider partial sum rules, i.e., limit the integrals to some maximum frequency. This approach is most promising when the valence electrons are well separated energetically from core electron states. Then, the high-energy part of the valence electron contribution to dielectric functions is still well defined and corresponds to a valence electron plasma frequency

$$\omega_{p,\mathrm{valence}}^2 = \frac{4\pi n_v e^2}{m_c}\,. \tag{4.22}$$

Here, n_v is the valence electron density and m_c is a mass which characterizes the electron mobility and is renormalized by polarization effects. The polarizability of ion cores also causes an almost frequency-independent shift of the dielectric function so that the intermediate asymptotic form of the dielectric function reads $\epsilon(\omega) \approx \epsilon_{\mathrm{core}} + \omega_{p,\mathrm{valence}}^2/\omega^2$.

4.1.3 Experiments

In this subsection, we show experimental results for the dielectric functions and the optical conductivity of the good metal aluminum. This material was chosen as an example due to the completeness of the available data which allows for the illustration of partial sum rules and for checking the universality of the full f-sum rule. Finally, we will see that photon experiments (reflection or transmission measurements) are superior over electronic energy-loss spectroscopy (EELS) for determining $\sigma(\omega)$.

Figure 4.2 a shows reflectance data for Al in the valence electron energy range up to $\hbar\omega = 22$ eV. The reflectivity is asymptotically nearly perfect at very small frequencies (see Fig. 4.2 b), then remains large up to the far ultra violet (UV) range with a dip at about $\hbar\omega \approx 1.5$ eV and decreases quite abruptly at $\hbar\omega \approx 15$ eV. By Kramers-Kronig analysis of this data, Ehrenreich et al. (1963) computed $\epsilon(\omega)$ shown in Fig. 4.2 c and $\sigma(\omega)$ as well as the loss function shown in Fig. 4.2 d. Here, the metallic behavior is visible as a Drude peak in $\mathrm{Re}\,\sigma(\omega)$ (not fully shown) and a divergence of $\mathrm{Im}\,\epsilon(\omega)$. The dip in the reflectance finds correspondence in a two-peak structure of