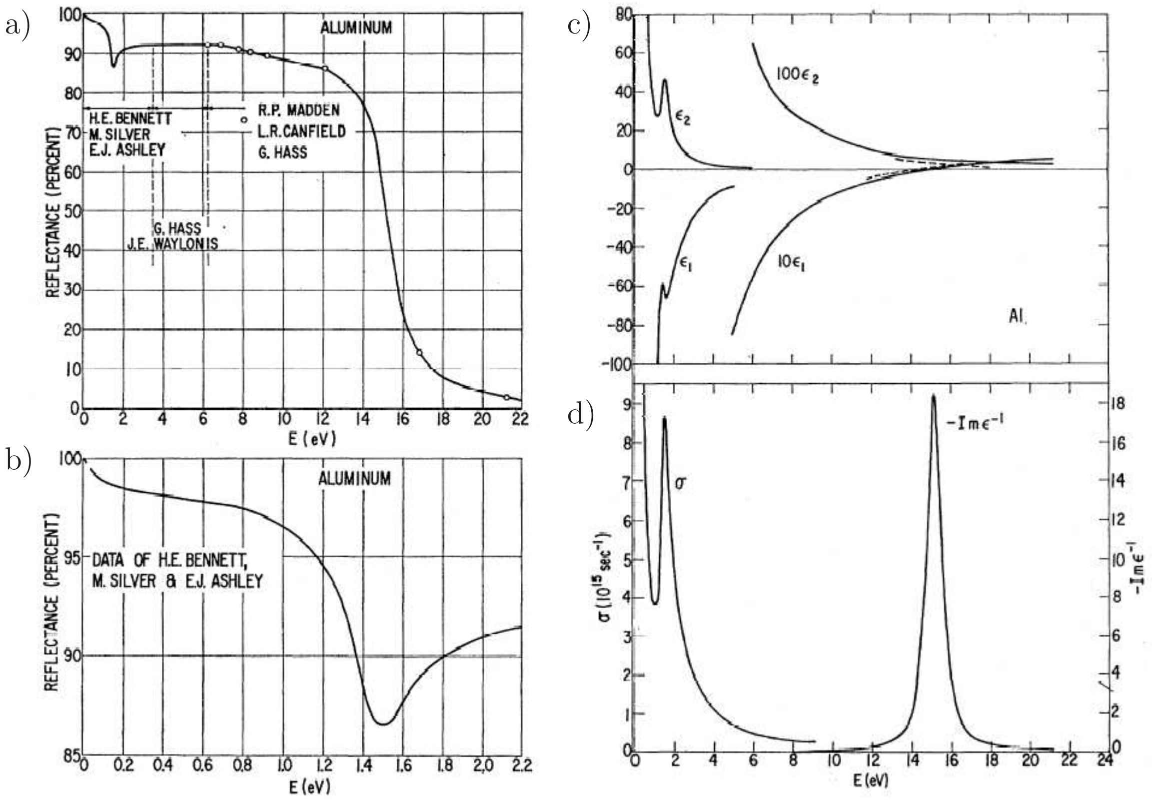

Figure 4.2: Low-energy optical properties of aluminum: **a)+b)** measured specular reflectivity; **c)** corresponding dielectric function $\epsilon(\omega) = \epsilon_1(\omega) + i\epsilon_2(\omega)$. **d)** Optical conductivity $\sigma(\omega)$ and loss function $-\mathrm{Im}\,\epsilon^{-1}(\omega)$ (Ehrenreich et al., 1963).

$\sigma(\omega)$. The electron energy-loss function $-\mathrm{Im}\,\epsilon^{-1}(\omega)$ is peaked at $\hbar\omega \approx 15$ eV which is close to the free valence-electron plasma frequency $\hbar\omega^0_{p,\mathrm{valence}} \approx 15.8$ eV. Hence, (4.22) applies for three valence electrons per atom and an only weakly renormalized mass $m_c \approx 1.05m$.

As an alternative to defining renormalized electron masses for valence electrons as in (4.22) one may use the bare mass and introduce effective electron densities. This is even more physical since the total weight is always proportional to the full electron density; consequently, the polarizability of core electrons only shifts the contribution of valence electrons to higher energies without modifying their total integral. Rescaling the integrands in (4.18)-(4.20) so that the total sum equals the number of electrons per atom, the weight up to some finite frequency may then be regarded as the effective number of electrons for that frequency range. As seen in Fig. 4.3 a, the partial sums deviate strongly from each other in and near the low frequency absorption range $\hbar\omega \lesssim 15$ eV. Since Al is nearly free-electron like, the effective number corresponding to (4.18) (indicated as "ϵ_2") saturates very close to 3, the true valence electron number, while those corresponding to (4.19) and (4.20) are reduced by a factor of $1/\sqrt{\epsilon_{\mathrm{core}}}$ and $1/\epsilon_{\mathrm{core}}$, respectively. At $\hbar\omega \approx 1$ keV, the effective numbers reach 11 while above 10 keV the sum rules correctly measure a total of 13