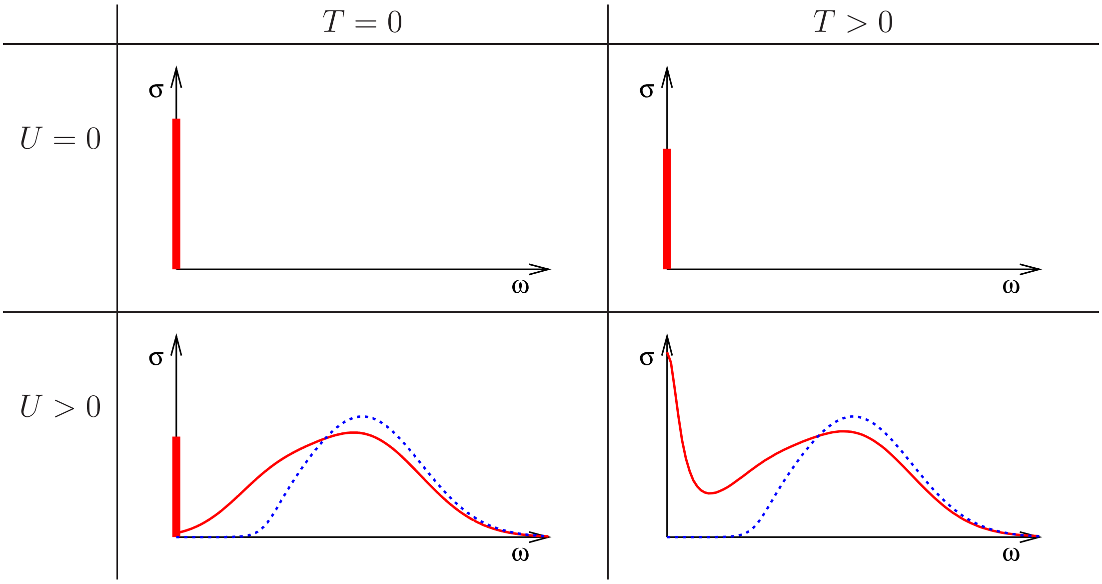

Figure 4.5: Scheme: qualitative behavior of the optical conductivity in Hubbard type models. For $U > 0$, one may distinguish metals (solid lines) and insulators (dashed lines). The thick vertical bars at $\omega = 0$ indicate δ-functions.

velocity $\boldsymbol{v}_{\boldsymbol{k},\nu}$ read[8] for a noninteracting dispersion $\epsilon_{\boldsymbol{k},\nu}$

$$\hat{\jmath} = \sum_{\boldsymbol{k},\nu,\sigma} e\, \boldsymbol{v}_{\boldsymbol{k},\nu}\, \hat{n}_{\boldsymbol{k},\nu,\sigma}; \quad \boldsymbol{v}_{\boldsymbol{k},\nu} = \frac{1}{\hbar} \boldsymbol{\nabla}_{\boldsymbol{k}}\, \epsilon_{\boldsymbol{k},\nu} \,. \tag{4.23}$$

The important observation which we are heading for is that due to the absence of dissipation no finite-frequency response arises in the noninteracting limit: In absence of electron-electron interaction, $\hat{H} = \hat{H}_0 = \sum_{\boldsymbol{k},\nu,\sigma} \epsilon_{\boldsymbol{k},\nu} \hat{n}_{\boldsymbol{k},\nu,\sigma}$. We thus have

$$[\hat{\jmath}, \hat{H}] = e \sum_{\boldsymbol{k},\nu,\sigma} \boldsymbol{v}_{\boldsymbol{k},\nu} [\hat{n}_{\boldsymbol{k},\nu,\sigma}, \hat{H}_0] = 0 \tag{4.24}$$

$$\Rightarrow \quad \hat{\jmath}(t) = \mathrm{e}^{i\hbar \hat{H} t}\, \hat{\jmath}(0)\, \mathrm{e}^{-i\hbar \hat{H} t} = \hat{\jmath}(0) \tag{4.25}$$

$$\Rightarrow \quad \sigma(\omega) = D\delta(\omega) \,, \tag{4.26}$$

where $D > 0$ is the Drude weight. The diamagnetic current could be neglected for this argument, since its contribution at finite frequencies is purely imaginary. Equation (4.26) remains valid for static mean-field theories of the density type, i.e., the Hartree approximation as long as translational invariance is retained. Thus, even in this case, $\sigma(\omega)$ is reduced to a Drude peak (of zero width). This general behavior is illustrated in Fig. 4.5, where we also distinguish between zero and finite temperature. Insulating behavior (dashed lines) is possible only for $U > 0$. Note that the total weight is in general temperature-dependent. Later we will see that some models corresponding to a semi-elliptic Bethe DOS do not yield a Drude peak in the noninteracting limit since their current operator and their kinetic energy operator do not commute.

[8] A more thorough introduction of the formalism follows in Sec. 4.2.