direction α are given by (lattice constant a)[16]

$$\hat{j}_\alpha = -\frac{ieta}{V\hbar}\sum_{i,\sigma}\left(\hat{c}^\dagger_{i\sigma}\hat{c}_{i+\boldsymbol{e}_\alpha\sigma} - \hat{c}^\dagger_{i+\boldsymbol{e}_\alpha\sigma}\hat{c}_{i\sigma}\right), \tag{4.40}$$

$$\hat{K}^0_\alpha = -t\sum_{i,\sigma}\left(\hat{c}^\dagger_{i\sigma}\hat{c}_{i+\boldsymbol{e}_\alpha\sigma} + \hat{c}^\dagger_{i+\boldsymbol{e}_\alpha\sigma}\hat{c}_{i\sigma}\right). \tag{4.41}$$

The full current density operator is obtained by variation of $\hat{K}$ with respect to the vector potential (Scalapino et al., 1992),

$$\hat{J}_\alpha = -\frac{c}{V}\frac{\delta\hat{K}}{\delta A_\alpha} = \hat{j}_\alpha + \frac{e^2a^2}{Vc\hbar^2}\hat{K}^0_\alpha A_\alpha. \tag{4.42}$$

As in the continuum case, the conductivity can be expressed in terms of a correlation function and of the diamagnetic contribution,

$$\sigma_{\alpha\alpha}(\omega) = \frac{iV}{\omega}\langle\langle\hat{j}_\alpha,\hat{j}_\alpha\rangle\rangle\,(\omega+i0^+) - i\frac{e^2a^2}{V\hbar^2(\omega+i0^+)}\langle\hat{K}^0_\alpha\rangle \tag{4.43}$$

Evidently, the reduction to a single-band model has changed the diamagnetic contribution and, consequently, the sum-rule to a nonuniversal form. For the assumption used in deriving (4.43) of nearest-neighbor hopping on a hypercubic lattice, the frequency sum is seen to be proportional to the kinetic energy,

$$\int_0^\infty \mathrm{d}\omega\,\sigma_{\alpha\alpha}(\omega) = -\frac{\pi}{2}\frac{e^2a^2}{V\hbar^2}\langle\hat{K}^0_\alpha\rangle = -\frac{\pi e^2}{2d\hbar^2}\,a^{2-d}\,\frac{\langle\hat{K}^0\rangle}{N}\,. \tag{4.44}$$

For the more general case of arbitrary hopping on a Bravais lattice, we have to consider that the phase shift due to the vector potential $\boldsymbol{A}$ increases with hopping distance. For later use, we want to obtain expressions in momentum representation. Thus, we assume for the unperturbed Hamiltonian (with band index ν),

$$\hat{H} = \sum_{\boldsymbol{k},\nu,\sigma}\epsilon_{\boldsymbol{k},\nu}\,\hat{n}_{\boldsymbol{k},\nu,\sigma} + \hat{H}_{\mathrm{ee}}\left\{\hat{n}_{i,\nu,\sigma}\right\}, \tag{4.45}$$

where the interaction term is of the density-density type.[17] Due to inversion symmetry we can always write for a selected lattice direction x,

$$\epsilon_{\boldsymbol{k},\nu} = \sum_{n=1}^{\infty} f_{\nu,n}(\boldsymbol{k}_\perp)\cos(na_xk_x) \tag{4.46}$$

[16]Note that for this particular case and up to prefactors, the kinetic energy and the current operator are sums over the real and the imaginary part, respectively, of the same bond hopping operator.

[17]Note that, in contrast to the continuum formulation, the restriction to density-density type interaction terms is in general a serious approximation in the Wannier representation; see subsection 1.1.3.