Then, to leading order,

$$\tilde{\rho}_{xx}(\epsilon) = \frac{1}{N} \sum_{\boldsymbol{k}} v_{k_x}^2 \delta(\epsilon - \epsilon_{\boldsymbol{k}_\perp}^\perp) = \rho(\epsilon)\, \frac{a_x}{2\pi} \int\limits_{-\pi/a_x}^{\pi/a_x} \mathrm{d}k_x\, v_{k_x}^2 \,. \tag{4.75}$$

Thus, expression (4.73) directly applies to all stacked lattices in $d \to \infty$ when hopping in current direction (lattice spacing a_x) is restricted to nearest neighbors with amplitude $t^*/\sqrt{Z}$. The suitable generalization for arbitrary-range hopping in current direction follows from (4.75). Note that the definition of hopping elements in current direction does not impact any local properties and that the frequency sum over $\sigma_{xx}(\omega)$ will be directly related to local properties (such as the kinetic energy) only for isotropic models.

4.3.2 *f*-sum Rule within the DMFT

In this subsection, we will derive a very general new form of the f-sum rule for the optical conductivity which applies to all cases where expression (4.65) for $\sigma(\omega)$ is used, in particular to $d = \infty$. We do not need to make any assumptions about the functional form of $\tilde{\rho}_{xx}(\epsilon)$ other than that it has to be piecewise differentiable. In particular, we do not need to know the dispersion $\epsilon_{\boldsymbol{k}}$; thus, our results can even be applied to lattices without (full) $\boldsymbol{k}$-space. Furthermore, we can show that the sum rule is fulfilled also for approximate DMFT solutions as long as the self-energy is causal. Finally, we present an efficient computation scheme for the frequency sum directly on the imaginary axis.

We start with the noninteracting limit. Then the spectral function becomes a δ-function, $A_\epsilon(\omega) \to \delta(\omega - \epsilon)$, and we obtain from (4.65) [with σ_0 defined in (4.67)],

$$\begin{aligned}
\int\limits_0^\infty \mathrm{d}\omega\, \sigma_{xx}(\omega) &= \frac{\sigma_0}{2} \int\limits_{-\infty}^\infty \mathrm{d}\epsilon\, \tilde{\rho}_{xx}(\epsilon) \int\limits_{-\infty}^\infty \mathrm{d}\omega \int\limits_{-\infty}^\infty \mathrm{d}\omega'\, \delta(\omega' - \epsilon)\delta(\omega + \omega' - \epsilon) \frac{n_\mathrm{f}(\omega') - n_\mathrm{f}(\omega' + \omega)}{\omega} \\
&= -\frac{\sigma_0}{2} \int\limits_{-\infty}^\infty \mathrm{d}\epsilon\, \tilde{\rho}_{xx}(\epsilon) \frac{\mathrm{d}}{\mathrm{d}\epsilon} n_\mathrm{f}(\epsilon) \;=\; \frac{\sigma_0}{2} \int\limits_{-\infty}^\infty \mathrm{d}\epsilon\, \tilde{\rho}'_{xx}(\epsilon)\, n_\mathrm{f}(\epsilon) && (4.76) \\
&= \frac{\sigma_0}{4} \left\langle \frac{\tilde{\rho}'_{xx}(\epsilon)}{\rho(\epsilon)} \right\rangle . && (4.77)
\end{aligned}$$

Here and in the following, the prime denotes the derivative with respect to the argument, $\tilde{\rho}'_{xx}(\epsilon) := \frac{\mathrm{d}}{\mathrm{d}\epsilon} \tilde{\rho}_{xx}(\epsilon)$. No boundary terms arise in the partial integration in (4.76) since $\tilde{\rho}(\epsilon)$ vanishes at the band edges. We further introduced the notation

$$\langle f(\epsilon) \rangle := \int\limits_{-\infty}^\infty \mathrm{d}\epsilon\, \rho(\epsilon)\, (n_{\epsilon,\uparrow} + n_{\epsilon,\downarrow})\, f(\epsilon) \,, \tag{4.78}$$

where the expectation value of the generalized momentum distribution function is for arbitrary interaction given in terms of Fermi distribution function $n_\mathrm{f}(\omega)$, chemical