Freericks, 1998*b*). After noting a numerical violation of the corresponding f-sum rule (4.44), i.e., a lack of proportionality between the integral over $\sigma(\omega)$ and the full kinetic energy, they realized the need for a careful treatment of the linear response formalism applicable to the Bethe lattice. They proposed (Chung and Freericks, 1998*a*)[24] an application of (4.73), but using a DOS multiplied by a nontrivial (squared) Fermi velocity prefactor of the form $\langle v_{\boldsymbol{k}_x}^2 \rangle(\epsilon) = 4 - \epsilon^2$, i.e., $\tilde{\rho}(\epsilon) = (4 - \epsilon^2)\rho(\epsilon)$. This prefactor vanishes at the band edges (at $\epsilon = \pm 2$) as expected on general physical grounds. Furthermore, the proportionality between the f-sum and the total kinetic energy was seen to be reestablished, albeit with an unexpected prefactor 3. No derivations were published; Freericks did, however, make a sketch of his treatment available privately (Freericks, 2000) which forms the basis of our derivation given below.

Let us first show analytically which sum rule obtains from using the hc decoupling $\tilde{\rho}_{xx}(\epsilon) = \langle v_{\boldsymbol{k}_x}^2 \rangle \rho(\epsilon)$ with $\rho(\epsilon)$ being the semi-elliptic Bethe DOS (and effectively constant $\langle v_{\boldsymbol{k}_x}^2 \rangle$). This is easily accomplished using the formalism developed in subsection 4.3.2,

$$\tilde{\rho}_{xx}(\epsilon) = \langle v_{\boldsymbol{k}_x}^2 \rangle \rho(\epsilon); \quad \rho(\epsilon) = \frac{1}{2\pi}\sqrt{4-\epsilon^2}$$

$$\Rightarrow \quad \frac{\mathrm{d}}{\mathrm{d}\epsilon}\tilde{\rho}_{xx}(\epsilon) = -\langle v_{\boldsymbol{k}_x}^2 \rangle \frac{\epsilon}{2\pi}\left(4-\epsilon^2\right)^{-1/2} \tag{4.96}$$

$$\Rightarrow \quad \int_0^\infty \mathrm{d}\omega\, \sigma_{xx}(\omega) = \langle v_{\boldsymbol{k}_x}^2 \rangle \frac{\sigma_0}{4} \langle \frac{-\epsilon}{4-\epsilon^2} \rangle \,. \tag{4.97}$$

Evidently, the frequency sum is indeed not proportional to the expectation value of the kinetic energy. This fact is not surprising since the proportionality of the f-sum to the kinetic energy is very specific to the hc case as shown in subsection 4.3.1.

The central assumption made by Freericks is that kinetic energy eigenstates can be written in the following form,[25]

$$|\epsilon\rangle = \sum_x (\gamma_\epsilon)^x \sum_{\alpha_x} |x, \alpha_x\rangle \,. \tag{4.98}$$

Here, the integer x counts the level number (i.e., is the vertical coordinate in Fig. 4.6 a) and α_x is an intra-level coordinate, i.e., labels lattice sites within level x. The latter is written with index x since the set of values that the intra-level coordinate can take depends on x for $Z > 2$. The factor[26] γ_ϵ takes the role of a phase shift between levels although, as we will see, its absolute value does not in general equal 1. An expression for γ_ϵ in terms of ϵ is obtained from the eigenvalue equation for the kinetic energy.

[24] Note that this reference is the published version of the preprint (Chung and Freericks, 1998*b*).

[25] Our reproduction of the derivation differs considerably in form from the sketch made available to us (Freericks, 2000). In particular, we found it necessary to introduce indices within the planes. Unless noted, however, the treatments are equivalent.

[26] In the following, we will also write $\gamma_\epsilon^x \equiv (\gamma_\epsilon)^x$ where x is still an exponent rather than an index.