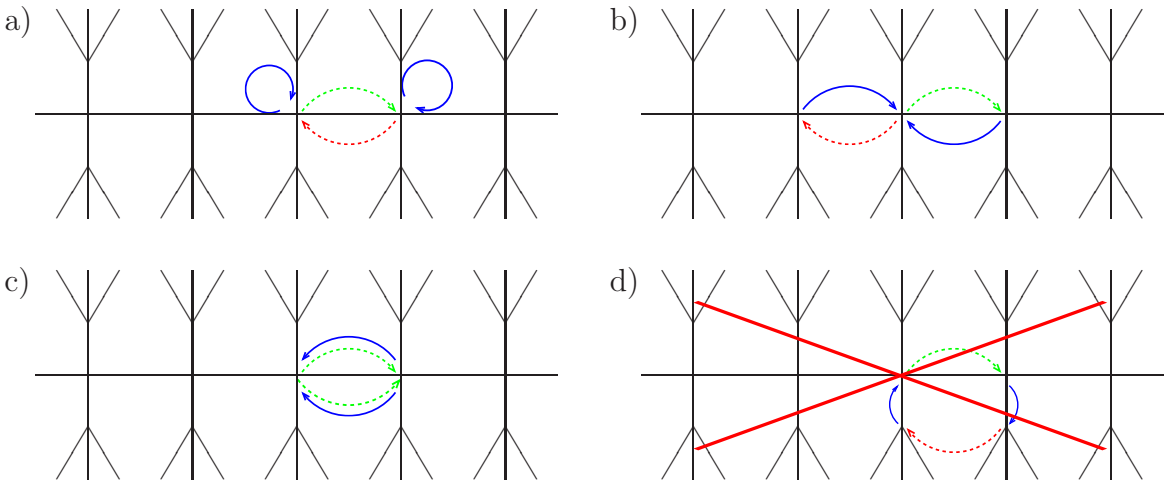

Figure 4.8: Diagrammatic representation of terms of the form (4.119) entering the computation of the optical conductivity: **a)** Leading contribution scaling as $\mathcal{O}(1/Z)$. **b)** and **c)** subleading contributions being $\mathcal{O}(1/Z^2)$. **d)** Contributions which include loops do not exist for the Bethe lattice (but are among the leading diagrams for regular lattices).

Here, the upper index 0 of operators stands for $\alpha = 0$, the central site within a sublattice. Consequently, the paramagnetic contribution to the optical conductivity

$$\sigma_{xx}(\omega) = -\frac{e^2a^2t^2}{i\hbar^2(\omega+i0^+)N}\sum_{i,j,\sigma}\langle\langle \hat{c}^{0\dagger}_{i+1,\sigma}\hat{c}^{0}_{i,\sigma} - \hat{c}^{0\dagger}_{i,\sigma}\hat{c}^{0}_{i+1,\sigma}, \hat{c}^{0\dagger}_{j+1,\sigma}\hat{c}^{0}_{j,\sigma} - \hat{c}^{0\dagger}_{j,\sigma}\hat{c}^{0}_{j+1,\sigma}\rangle\rangle\,(\omega+i0^+) \tag{4.118}$$

contains terms of the form (with suppressed ω-dependence)

$$\Gamma_{ijkm} := \langle\langle \hat{c}^{0\dagger}_{i\sigma}\hat{c}^{0}_{k\sigma}, \hat{c}^{0\dagger}_{m\sigma}\hat{c}^{0}_{j\sigma}\rangle\rangle\,\delta_{i,k\pm1}\delta_{m,j\pm1}\,. \tag{4.119}$$

In a diagrammatic evaluation of the corresponding real-time correlation function (cf. Sec. D.2), the exponentials carrying the time dependence in the Heisenberg picture introduce additional paths between pairs of sites; the resulting full diagram only contributes to the implicit trace when it represents a closed path. A selection of such diagrams is shown in Fig. 4.8 where the current paths[35] (dashed lines) are closed by time-dependent propagators (solid lines). In the DMFT formalism, each path of taxicab length l carries a factor of $Z^{-l/2}$ so that the diagrams in Fig. 4.8a, Fig. 4.8b, and Fig. 4.8c, have prefactors of $1/Z$, $1/Z^2$, and $1/Z^2$, respectively. Due to the absence of loops, the additional powers of $1/Z$ introduced by nonlocal propagators cannot be compensated by factors of Z arising from a sum over equivalent diagrams. Thus, up to a shift along the current direction, diagram Fig. 4.8a is the only (leading) contribution for $Z \to \infty$. For comparison, a diagram typical of regular lattices is shown in Fig. 4.8d; such diagrams usually contribute in leading order Z/Z^2, but do not exist on the Bethe lattice. The two time-dependent propagators closing the path do not interfere for $Z \to \infty$ on any lattice (see subsection 4.4.4) which implies that

[35] In momentum representation, these "current vertices" contract to a pair of dots.