the expectation values decouple and lead to a pair of single-particle Green functions, i.e., to the usual bubble sum. In the specific case under consideration, i.e., without loops, both Green functions become local, so that the conductivity reads

$$\sigma_{xx}(\omega) = \frac{\pi e^2 a^2 t^2}{\hbar^2} \frac{L_x}{L} \sum_{\sigma} \int_{-\infty}^{\infty} \mathrm{d}\omega' A_{A\sigma}(\omega') A_{B\sigma}(\omega' + \omega) \frac{n_{\mathrm{f}}(\omega') - n_{\mathrm{f}}(\omega' + \omega)}{\omega} . \quad (4.120)$$

Here, $A(\omega)$ denotes the full local spectral function, L_x the linear extent in current direction, and L the number of sites for some finite part of the system. As indicated by the sublattice indices A, B and spin index σ, the result is valid also in phases with broken symmetry, i.e., in an antiferromagnetic phase. Note that (4.120) is not directly dependent on the type of the underlying sublattice (confined in each hyperplane), not even via a noninteracting DOS.[36]

The most striking feature of the present derivation and its result (4.120) is that the dc conductivity always remains finite (or vanishes) in the paramagnetic phase, even in the noninteracting limit. This follows from the fact that in the paramagnetic phase, the local spectral function is bounded by the maximum of the noninteracting DOS. For a symmetric DOS which is peaked at zero frequency, the integral in (4.120) is therefore bounded by $\rho^2(0)$.[37] Consequently, there is no qualitative difference between the conductivities for noninteracting and interacting systems: only the energy eigenvalues of each pair of states determines its contribution to $\sigma_{xx}(\omega)$; the associated momenta are unimportant. This behavior is in striking contrast to the Bravais lattice case discussed in subsection 4.1.4 where the dc conductivity diverges in the noninteracting limit. In this sense, (4.120) describes incoherent transport. As we will show in subsection 4.5.1, the expression is generally valid within the DMFT for $\boldsymbol{k}_\perp$-nonconserving hopping between planes.

Commutators and Loops

As discussed in subsection 4.1.4, the presence of optical absorption at finite frequencies in the noninteracting limit implies that the paramagnetic current operator and the kinetic energy do not commute. In the following, we will demonstrate that indeed $[\hat{H}_0, \hat{\jmath}] \neq 0$ for the single-chain layout discussed in this subsection. For this purpose, we focus first on a single contribution to the current operator and study separately and in turn its commutator with kinetic energy contributions (anti)parallel or perpendicular to its direction. We will see that the former contributions vanish quite generically (in absence of disorder) in summations along the current chains. In contrast, the latter contributions can only cancel in the presence of certain loops of length four which do not exist on the Bethe lattice.

For a single hopping term contributing to the current operator, the commutator

[36] In subsection 4.5.1, we will present a unified treatment of incoherent and coherent transport where $\boldsymbol{k}$-sums and noninteracting DOSs reappear for the case of regular sublattices.

[37] In contrast, momentum-dependent spectral functions diverge in the noninteracting limit.