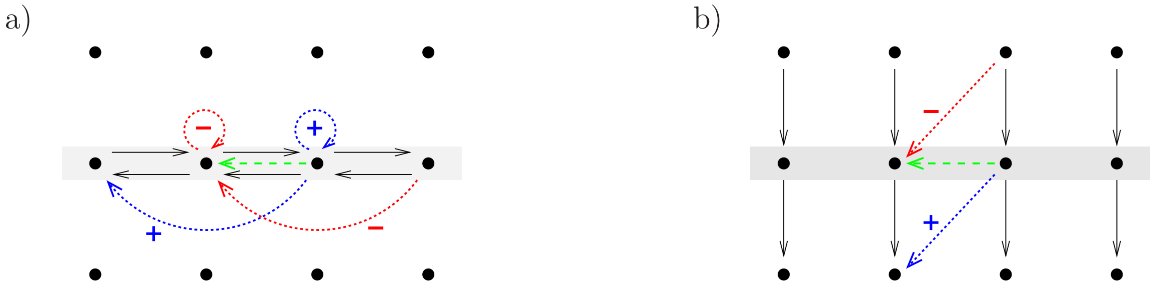

Figure 4.9: Commutator of kinetic energy (solid arrows) and one paramagnetic current term (dashed arrow). **a)** Contributions from hopping along the current lead to on-site terms and next-nearest neighbor hopping terms in the commutator (dotted arrows) vanish in sums along the current chain. **b)** Perpendicular hopping induces NNN hopping terms (dotted arrows) away from and towards the current chain (shaded).

$[\hat{H}_0, \hat{\jmath}]$ contains terms from hopping in current direction,

$$\begin{aligned}\sum_i \left[\hat{c}_i^{0\dagger}\hat{c}_{i+1}^0, \hat{c}_j^{0\dagger}\hat{c}_{j+1}^0\right] &= \sum_i \left[\hat{c}_i^{0\dagger}\big(\delta_{i+1,j}^0 - \hat{c}_j^{0\dagger}\hat{c}_{i+1}^0\big)\hat{c}_{j+1} - \hat{c}_j^{0\dagger}\hat{c}_{j+1}^0\,\hat{c}_i^{0\dagger}\hat{c}_{i+1}^0\right] \\ &= \hat{c}_{j-1}^{0\dagger}\hat{c}_{j+1}^0 - \sum_i \left[\hat{c}_j^{0\dagger}\hat{c}_i^{0\dagger}\hat{c}_{j+1}^0\hat{c}_{i+1}^0 + \hat{c}_j^{0\dagger}\hat{c}_{j+1}^0\,\hat{c}_i^{0\dagger}\hat{c}_{i+1}^0\right] \\ &= \hat{c}_{j-1}^{0\dagger}\hat{c}_{j+1}^0 - \hat{c}_j^{0\dagger}\hat{c}_{j+2}^0 \end{aligned} \tag{4.121}$$

and from hopping opposite to the current direction,

$$\sum_i \left[\hat{c}_{i+1}^{0\dagger}\hat{c}_i^0\ , \hat{c}_j^{0\dagger}\hat{c}_{j+1}^0\right] = \hat{c}_{j+1}^{0\dagger}\hat{c}_{j+1}^0 - \hat{c}_j^{0\dagger}\hat{c}_j^0\ . \tag{4.122}$$

Both the generating and the generated terms for the parallel and antiparallel cases are shown schematically in Fig. 4.9 a. Evidently, they both cancel in a sum over the full current,

$$\sum_{i,j} \left[\hat{c}_i^{0\dagger}\hat{c}_{i+1}^0, \hat{c}_j^{0\dagger}\hat{c}_{j+1}^0\right] = 0 = \sum_{i,j} \left[\hat{c}_{i+1}^{0\dagger}\hat{c}_i^0\ , \hat{c}_j^{0\dagger}\hat{c}_{j+1}^0\right] . \tag{4.123}$$

In contrast, hopping processes perpendicular to the current introduce "diagonal" terms in the commutator as depicted in Fig. 4.9 b,[38]

$$\sum_{\substack{i,\alpha \\ \alpha \text{ NN of } 0}} \left[\hat{c}_i^{\alpha\dagger}\hat{c}_i^0\ , \hat{c}_j^{0\dagger}\hat{c}_{j+1}^0\right] = \sum_{\substack{\alpha \\ \alpha \text{ NN of } 0}} \hat{c}_j^{\alpha\dagger}\hat{c}_{j+1}^0\, , \tag{4.124}$$

which can only vanish in summations over parallelogram loops of length 4 which are not present in the model under consideration. For the hypercubic case of a regular lattice, the cancellation is diagrammatically represented in Fig. 4.10. Consequently,

[38]Within the current notation, also terms of the form $\hat{c}_i^{0\dagger}\hat{c}_i^{\alpha}$ contribute to the kinetic energy. Fig. 4.9 b shows a selection of both contributions.