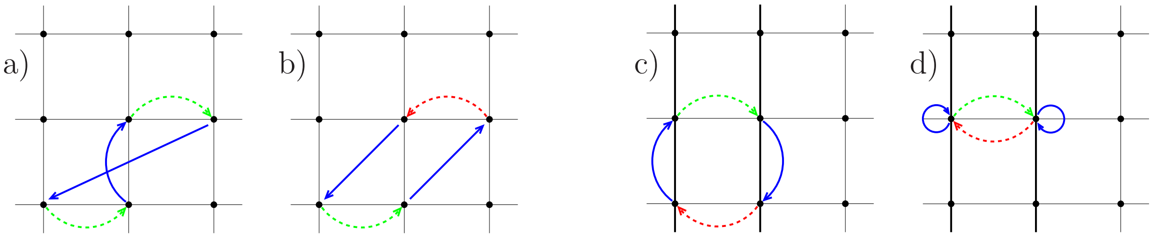

Figure 4.13: Diagrammatic contributions to the current-current correlation function (cf. Fig. 4.8). Here, the vertical axis symbolizes all directions perpendicular to the current direction (horizontal axis). In finite dimensions, contributions arise for **a)** parallel and **b)** antiparallel current (dashed arrows) with arbitrary offset. In $d \to \infty$, however, only the antiparallel contribution survives with no offset in current direction, **c)**+**d)**. The connecting Green functions (solid arrows) are, then, restricted to two neighboring planes.

the horizontal axis is the current direction whereas the vertical axis represents all other (perpendicular) directions. Diagrams as shown in Fig. 4.13 a and Fig. 4.13 b which include sites at more than 2 planes are suppressed in high dimensions where only diagrams like shown in Fig. 4.13 c and Fig. 4.13 d survive. The reason is that each hopping in current direction contributes a factor that scales to zero in the DMFT limit (and is proportional to $1/\sqrt{Z}$ in the isotropic case) while all other directions are inequivalent by definition so that these factors cannot cancel from a sum over equivalent diagrams. As a consequence, the time evolution paths (solid lines) between the current vertices are each restricted to a separate hyperplane and cannot interfere which implies that the full correlation function can be expressed in terms of a pair of Green functions (Metzner et al., 1992). In more formal terms, when we label the involved sites as A_1 and A_2 for one (hyper)plane and B_1, B_2 for the second plane then the expectation value of the time-dependent current-current correlation function decomposes in the following way:

$$\langle e^{-i\hat{H}t} c^\dagger_{A_1} c_{B_1} e^{i\hat{H}t} c^\dagger_{B_2} c_{A_2} \rangle = \langle e^{-i\hat{H}t} c_{B_1} e^{i\hat{H}t} c^\dagger_{B_2} \; c_{A_2} e^{-i\hat{H}t} c^\dagger_{A_1} e^{i\hat{H}t} \rangle \tag{4.129}$$

$$= \langle e^{-i\hat{H}t} c_{B_1} e^{i\hat{H}t} c^\dagger_{B_2} \rangle \, \langle c_{A_2} e^{-i\hat{H}t} c^\dagger_{A_1} e^{i\hat{H}t} \rangle \tag{4.130}$$

$$= G_{B_1 B_2}(t) G_{A_2 A_1}(-t)\,. \tag{4.131}$$

Here, we have used the invariance of the implicit traces under ring permutations and the time invariance of the Hamiltonian. Thus, this real-space argument implies a re-derivation of the bubble formula (4.62) for $\sigma(\omega)$ in $d \to \infty$. In contrast to the $\boldsymbol{k}$-space formulation presented in Sec. 4.3 for regular lattices, the real-space formulation applies to arbitrary (pseudo-) lattices and even in the presence of disorder.

Full diagonal disorder is realized within the DMFT when all hopping matrix elements are chosen independently from a random distribution with $\langle t_{ij} \rangle = 0$ (cf. subsection 2.2.2). Then, only diagrams which are self-retracing in real space have finite expectation values. Taken together with the restriction to diagrams living on two neighboring planes, this means that the current-current correlation function becomes fully local, i.e., reduces to contributions of the form $\langle\langle \hat{c}^\dagger_{\boldsymbol{r}} \hat{c}_{\boldsymbol{r}+\boldsymbol{e}_x}, \hat{c}^\dagger_{\boldsymbol{r}+\boldsymbol{e}_x} \hat{c}_{\boldsymbol{r}} \rangle\rangle$.