Therefore, the Green functions appearing in the bubble formula for $\sigma(\omega)$ are local in the current direction. Let us for the moment specialize in the general stacked case where the hyperplanes are connected by nearest-neighbor bonds in current direction which is assumed to be perpendicular to the hyperplanes. If we consider the homogeneous case and denote the real space Green function connecting site $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ as $G_{\boldsymbol{r}_2-\boldsymbol{r}_1}(\omega)$, the resulting expression contributing to the conductivity is of the form

$$\sum_{\boldsymbol{r}_\perp} G_{(0,\boldsymbol{r}_\perp)}(\omega)\; G_{(0,-\boldsymbol{r}_\perp)}(\omega')\,. \tag{4.134}$$

The important point is that the Green functions are local in current direction; $\boldsymbol{r}_\perp$ is the perpendicular component of $\boldsymbol{r}$. If the hyperplanes are regular lattices, this translates in Fourier space into a contribution

$$\sum_{\boldsymbol{k}}\sum_{\boldsymbol{k}'} G_{\boldsymbol{k}}(\omega)\, G_{\boldsymbol{k}'}(\omega')\, \delta(\boldsymbol{k}_\perp - \boldsymbol{k}'_\perp)\,. \tag{4.135}$$

This simple form applies to the hypercubic lattice and all "full" stacked lattices, where each lattice site is connected to neighboring sites by bonds in current direction. Compared with the finite-dimensional case where the bubble summation would have to include diagrams as shown in Fig. 4.13c and which would give rise to the full δ-function $\delta(\boldsymbol{k}-\boldsymbol{k}')$, the restriction of the δ-function to perpendicular directions might already be viewed as a reduction of coherence. Obviously the fully incoherent case is realized when there is only a single chain of links in current direction as discussed in subsection 4.4.2 or if the hopping in current direction is fully disordered as discussed in subsection 4.4.4: in both cases, the δ-function is replaced by unity.

The main point of this subsection is that the degree of coherence in $\boldsymbol{k}_\perp$ may be tuned for an anisotropic model by attaching bonds in current direction only to a sublattice of sites within each hyperplane (with the extreme cases where sublattice and full lattice within the hyperplane are identical or when only a single site per plane is connected). For example, when every second site (i.e., any site with coordinates $\boldsymbol{r}$ for which $(-1)^{||\boldsymbol{r}_\perp||}=1$) is connected, the transport is coherent up to the AF wave vector which leads to a contribution[43]

$$\sum_{\boldsymbol{k}}\sum_{\boldsymbol{k}'} G_{\boldsymbol{k}}(\omega)\, G_{\boldsymbol{k}'}(\omega')\, \frac{1}{2}\big(\delta(\boldsymbol{k}_\perp - \boldsymbol{k}'_\perp) + \delta(\boldsymbol{k}_\perp - \boldsymbol{k}'_\perp - \boldsymbol{Q}_\perp)\big)\,. \tag{4.136}$$

Finally, for a hc lattice (with arbitrary hopping range in the perpendicular direction) where every n_α^{th} site in direction α is part of the "current sublattice", the $\boldsymbol{k}$ summation reads

$$\sum_{\boldsymbol{k}}\sum_{\boldsymbol{k}'} G_{\boldsymbol{k}}(\omega)\, G_{\boldsymbol{k}'}(\omega')\, \prod_{\alpha=2}^{d}\left(\frac{1}{n_\alpha}\sum_{l=0}^{n_\alpha-1}\delta(k_\alpha - k'_\alpha - \frac{l}{n_\alpha}\frac{2\pi}{a_\alpha})\right)\,. \tag{4.137}$$

Here, a_α is the lattice spacing in direction α. The completely incoherent case is recovered for $n_\alpha \to \infty$ when all δ-functions approach unity.