will here derive the correct expressions, i.e., perform the necessary momentum sums and compare the results to an evaluation of the hc expressions.

In the low-frequency limit of the homogeneous phase, the transverse conductivity reduces to (Shastry, Shraiman, and Singh, 1993; Merino and McKenzie, 2000)[46]

$$\sigma_{xy} = \sigma_0^{\mathrm{H}} \int_{-\infty}^{\infty} \mathrm{d}\omega\, \frac{\partial n_f(\omega)}{\partial \omega} \frac{1}{N} \sum_{\boldsymbol{k}} \Big(\frac{\partial \epsilon_{\boldsymbol{k}}}{\partial k_x}\Big)^2 \frac{\partial^2 \epsilon_{\boldsymbol{k}}}{\partial k_y^2} A^3_{\epsilon_{\boldsymbol{k}}}(\omega)\,. \tag{4.140}$$

Here, $\sigma_0^{\mathrm{H}} = 4\pi^2 |e|^3 aB/3\hbar^2$ and B is the magnetic field in z direction. The spectral function $A_{\epsilon_{\boldsymbol{k}}}(\omega)$ depends on momentum $\boldsymbol{k}$ only via the free dispersion $\epsilon_{\boldsymbol{k}}$. The dc Hall coefficient

$$R_{\mathrm{H}} = \frac{\sigma_{xy}}{B\sigma_{xx}^2(\omega = 0)} \tag{4.141}$$

follows from (4.140) and the limit $\omega \to 0$ of (4.65). The thermopower is given by the expression (Schweitzer and Czycholl, 1991; Pruschke, Jarrell, and Freericks, 1995; Merino and McKenzie, 2000)

$$S = -\frac{k_B}{|e|T} \frac{L_{12}}{L_{11}} \tag{4.142}$$

where the transport integrals reduce for $d \to \infty$ to[47]

$$L_{ij} = -\int_{-\infty}^{\infty} \mathrm{d}\omega\, \frac{\partial n_f(\omega)}{\partial \omega} \Big[\frac{2}{N} \sum_{\boldsymbol{k}} \big(\frac{\partial \epsilon_{\boldsymbol{k}}}{\partial k_x}\big)^2 A^2_{\epsilon_{\boldsymbol{k}}}(\omega)\Big]^i \omega^{j-1} \tag{4.143}$$

In order to perform the sums over momenta, we will use the fact that the dispersion of the $t - t'$ lattice may be expressed in terms of the dispersion $\epsilon_{\boldsymbol{k}}^{\mathrm{hc}}$ of the hypercubic lattice with unit scaled hopping amplitude $t^* = 1$,

$$\epsilon(\boldsymbol{k}, t^*, a^*) \equiv \epsilon_{\boldsymbol{k}} = t^* \epsilon_{\boldsymbol{k}}^{\mathrm{hc}} + \frac{a^* t^*}{\sqrt{2}} \Big(1 - \big(\epsilon_{\boldsymbol{k}}^{\mathrm{hc}}\big)^2\Big) \equiv f(\epsilon_{\boldsymbol{k}}^{\mathrm{hc}})\,. \tag{4.144}$$

Consequently, we may write

$$v_{\boldsymbol{k},x} \equiv \frac{\partial \epsilon_{\boldsymbol{k}}}{\partial k_x} = t^* \big(1 - \sqrt{2}\, a^* \epsilon_{\boldsymbol{k}}^{\mathrm{hc}}\big) \frac{\partial \epsilon_{\boldsymbol{k}}^{\mathrm{hc}}}{\partial k_x} \tag{4.145}$$

where $\partial \epsilon_{\boldsymbol{k}}^{\mathrm{hc}}/\partial k_x = \sqrt{\frac{2}{d}} \sin(k_x)$. Averaging over k_x, this yields for the squared Fermi velocity

$$\begin{aligned} \langle v_{\boldsymbol{k},x}^2 \rangle(\epsilon) &= \frac{t^{*2}}{d} \Big(1 - \sqrt{2} a^* \tilde{\epsilon}\Big)^2 \Big|_{\epsilon = f(\tilde{\epsilon})} && (4.146) \\ &= \frac{1 + 2a^{*2} - 2\sqrt{2} a^* \sqrt{1 + a^{*2}}\epsilon}{1 + a^{*2}}\,. && (4.147) \end{aligned}$$

to avoid the cumbersome sums over momentum". They claim it to be justified by their focus "on many-body effects and not on how different band structures may change the results slightly".

[46] In finite dimensions, the second derivative in x direction is supplemented by a term $-\big(\frac{\partial \epsilon_{\boldsymbol{k}}}{\partial k_y}\big)^2 \frac{\partial^2 \epsilon_{\boldsymbol{k}}}{\partial k_x \partial k_y}$ (which carries an additional factor $1/d$ and, therefore, vanishes for $d \to \infty$).

[47] Note that this expression is only approximate since it does not take contributions due to the flow of doubly occupied sites into account (Georges et al., 1996).