Here, R_i (P_i) label the phonon position (momentum) at site i, g is a coupling strength, M is the phonon mass and Ω its frequency.[4]

Finally, the relativistic spin-orbit interaction can be approximately reintroduced in the form

$$\hat{H}_{SO} = \xi \sum_{\nu\nu'\sigma\sigma'} \langle \nu\sigma | \boldsymbol{L} \cdot \boldsymbol{S} | \nu'\sigma' \rangle c^\dagger_{\nu\sigma} c_{\nu'\sigma'}, \tag{A.10}$$

where L is the angular momentum operator, $\boldsymbol{S}$ the vector of Pauli matrices, and the parameter ξ depends on the system (see, e.g., Bruno, 1993; Harle, 2000). In the presence of magnetic order, this interaction may lead to an easy or hard axis. Within DMFT, the behavior of antiferromagnetic systems with a strong easy axis can also be modeled phenomenologically by restricting local spin moments to point along this axis which then leads to metamagnetic phase transitions of both first and second order upon variation of an external magnetic field (Held, Ulmke, and Vollhardt, 1996; Held, Ulmke, Blümer, and Vollhardt, 1997).

A.2 Characterization of Generic Momenta

In the following, we will first extend the definition of nongeneric momenta given in subsection 1.2.2 to finite dimensions, then show that in all dimensions nongeneric momenta can be characterized as being close to the origin $\mathbf{0}$ or to the antiferromagnetic wave vector $\boldsymbol{Q}$ and, finally, draw some conclusions for the $d \to \infty$ limit of the hypercubic lattice and the DMFT in general.

In the following, we will exploit that the expression $\eta_{\boldsymbol{q}} = \lim_{d\to\infty} \frac{1}{d} \sum_{\alpha=1}^{d} \cos(q_\alpha)$ introduced in the context of (1.23) for the $d = \infty$ hc lattice naturally extends to finite dimensions. Since $\eta_{\boldsymbol{q}}$ always varies in the range $[-1, 1]$ and $\eta_{\boldsymbol{q}} \neq 0$ defines nongeneric vectors $\boldsymbol{q}$ in $d = \infty$, one can identify vectors $\boldsymbol{q}$ with $|\eta_{\boldsymbol{q}}| > \eta_0$ (for any $\eta_0 > 0$) with strong momentum correlation in any finite dimension d and loosely call them nongeneric, too. Furthermore, we can estimate $\eta_{\boldsymbol{q}}$ from the vector norm $|\boldsymbol{q}|$ which for momenta in the first Brillouin zone, unit lattice spacing $a = 1$, and dimension d varies between $|\mathbf{0}| = 0$ and $|\boldsymbol{Q}| = \pi\sqrt{d}$:

$$\eta_{\boldsymbol{q}} = -1 + \frac{1}{d} \sum_{\alpha=1}^{d} (1 - \cos(\boldsymbol{q}_\alpha)) \leq \frac{|\boldsymbol{q}|^2}{2d} - 1 \tag{A.11}$$

Therefore, η is strictly negative for all momenta inside a hypersphere of radius $\sqrt{2d}$ while $\eta < -0.5$ for momenta $|\boldsymbol{q}| < \sqrt{d}$. Analogously, $\eta > 0$ ($\eta > 0.5$) for the corresponding hyperspheres around $\boldsymbol{Q}$. Thus, we have identified convex sets of "nongeneric" momenta illustrated in Fig. A.1 a which are associated with strong energetic correlations via (1.23) and which acquire $\epsilon_{\boldsymbol{q}} = -\infty$ ($\epsilon_{\boldsymbol{q}} = \infty$) in the limit $d \to \infty$ on the hc lattice. In Fig. A.1 b, we show the probability distribution of η_q for $d = 1, 2, 3$ as

[4] In principle, "phonon coordinate" can be translated as "ion coordinate" (which is d-dimensional). Note, however, that the single Einstein phonon mode considered here only has a single coordinate, i.e., is one-dimensional. A generalization to (anisotropic) d-dimensional coupling would require the inclusion of several phonon frequencies.