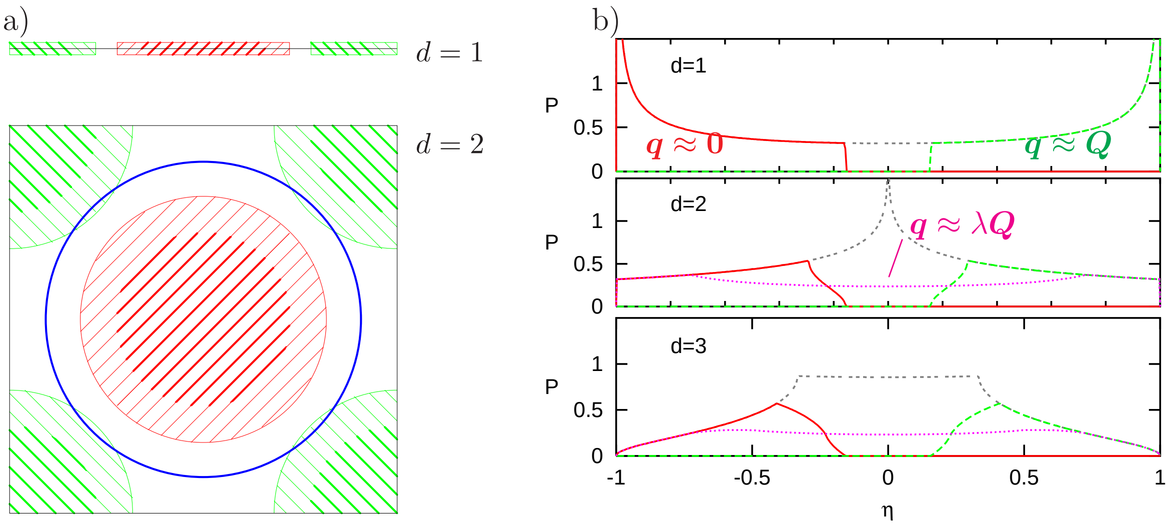

Figure A.1: a) Distribution of "nongeneric" momenta $\boldsymbol{q}$ for the hypercubic lattice in dimensions 1 and 2. Light (heavy) shading indicates states with $|\boldsymbol{q}| < \sqrt{2d}$ ($|\boldsymbol{q}| < \sqrt{d}$) which implies $|\eta_{\boldsymbol{q}}| > 0$ ($|\eta_{\boldsymbol{q}}| > 0.5$), when estimated from (A.11). The thick circle indicates the rescaled average length $\pi\sqrt{d/3}$ of vectors in $d \to \infty$. **b)** Full probability distribution $P(\eta_{\boldsymbol{q}})$ (rescaled hc DOS) for $d = 1, 2, 3$ and partial distributions corresponding to hyperspheres with $|\boldsymbol{q}| < \sqrt{2d}$ (solid lines) or $|\boldsymbol{q} - \boldsymbol{Q}| < \sqrt{2d}$ (dashed lines) (cf. shaded areas in a) or to momenta close to the diagonal (dotted lines).

well as partial distributions corresponding to the restricted set of momenta $|\boldsymbol{q}| < \sqrt{2d}$ and $|\boldsymbol{q}-\boldsymbol{Q}| < \sqrt{2d}$. While the criterion (A.11) is obviously not sharp, it clearly selects states with extremal values of $\eta_{\boldsymbol{q}}$ (and, for a hc lattice, also with extremal energy $\epsilon_{\boldsymbol{q}}$). For comparison, the partial probability distributions are also shown for momenta which are close to the diagonal extending from $\boldsymbol{0}$ to $\boldsymbol{Q}$. While this set of vectors with $\min_\lambda\{|\boldsymbol{q} - \lambda\boldsymbol{Q}|\} < \pi^2 d/12 - \pi^2\sqrt{d/180}$ (vectors one standard deviation closer to the diagonal than the average in the reduced Brillouin zone) has overlap with the other two sets for fixed (finite) dimension, it does not pick out extremal $\eta_{\boldsymbol{q}}$ and can therefore not be associated with nongeneric momenta.[5] We stress that the Euclidean radius of the selected hyperspheres is of the order of the longest vector within the Brillouin zone in all dimensions, i.e., is not small in high dimensions.

Therefore, transitions with small momentum transfer (measured by Euclidean length) imply small energy transfer in any dimension, also in the limit $d = \infty$. Consequently, the concept of a Fermi surface remains meaningful in this limit. The Fermi body of states occupied at low T even remains simply connected within the Brillouin zone in the noninteracting limit. Thus, energies are not random in momentum space.

[5] This finding contradicts statements made in Gebhard's book which we cite here for comparison: *"Furthermore, on the hypercubic lattice, some correlations remain in the nesting direction,* $\boldsymbol{q} = \lambda\boldsymbol{Q}$ *(*$|\lambda| \leq 1$*). For all other* $\boldsymbol{q}$ *with* $|\boldsymbol{q} - \lambda\boldsymbol{Q}| > \mathcal{O}(\sqrt{1/d})$*, however, the two energies are completely uncorrelated,..." (Gebhard, 1997).* While it is clear that strictly on the diagonal $\boldsymbol{q} = \lambda\boldsymbol{Q}$ (with $0 \leq \lambda \leq 1$) all values $\lambda \neq 0.5$ correspond to finite $\eta_{\boldsymbol{q}} = \cos(\lambda\pi)$, this is evidently not true for its surroundings.