Appendix B

Hyperdiamond Lattice

The hyperdiamond (hd) lattice is a bipartite non-Bravais lattice which generalizes the $d = 2$ honeycomb lattice and the $d = 3$ diamond lattice to arbitrary dimensions. In addition to the usual definition and recursive construction (Santoro, Airoldi, Sorella, and Tosatti, 1993) we will here use a representation in $d + 1$ dimensions (van Dongen, 1997). As we will show, only the latter clearly exposes the isotropy of the hyperdiamond lattice in the long-wavelength limit and allows for a computation of the optical conductivity not only in $d = \infty$ but, within the local approximation for self-energy Σ and particle-hole irreducible vertex Γ, also for arbitrary finite dimensions (see chapter 4). Although an extension to longer range hopping is straightforward we will in this section only consider NN hopping.

A general characterization of the hyperdiamond lattice in dimension $d \geq 1$ is (Santoro et al., 1993)

1. each lattice site has $d + 1$ nearest neighbors (NN),
2. the angle between each pair of $d+1$ unit[1] primitive lattice vectors $\boldsymbol{\tau}_l$ connecting a site to its NN is constant,
$$\boldsymbol{\tau}_l \cdot \boldsymbol{\tau}_m = \gamma_d, \tag{B.1}$$
3. for $d > 1$, there is no site-centered inversion symmetry, but only bond-centered inversion symmetry,

where the last statement follows from the first two. The lack of inversion symmetry implies that the unit vectors $\{\boldsymbol{\tau}_l\}$ cannot be the same on A and B sublattices. Instead they are related by an inversion operation, $\boldsymbol{\tau}_l^B = -\boldsymbol{\tau}_l^A$. Thus it is useful to rewrite the noninteracting Hamiltonian by explicitly separating A and B lattice sites,

$$\begin{aligned}
\hat{H}_0 &= -t \sum_{\langle i,j\rangle,\sigma} \left(\hat{c}^\dagger_{\boldsymbol{r}_i\sigma} \hat{c}_{\boldsymbol{r}_j\sigma} + \hat{c}^\dagger_{\boldsymbol{r}_i\sigma} \hat{c}_{\boldsymbol{r}_j\sigma} \right) \\
&= -t \sum_{i\in A,\sigma} \sum_{l} \hat{c}^\dagger_{\boldsymbol{r}_i+\boldsymbol{\tau}_l^A,\sigma} \hat{c}_{\boldsymbol{r}_i\sigma} - t \sum_{j\in B,\sigma} \sum_{l} \hat{c}^\dagger_{\boldsymbol{r}_j+\boldsymbol{\tau}_l^B,\sigma} \hat{c}_{\boldsymbol{r}_j\sigma} \\
&= -t \Big[\sum_{i\in A,\sigma} \sum_{l} \hat{c}^\dagger_{\boldsymbol{r}_i+\boldsymbol{\tau}_l,\sigma} \hat{c}_{\boldsymbol{r}_i\sigma} + \sum_{j\in B,\sigma} \sum_{l} \hat{c}^\dagger_{\boldsymbol{r}_j-\boldsymbol{\tau}_l,\sigma} \hat{c}_{\boldsymbol{r}_j\sigma} \Big],
\end{aligned} \tag{B.2}$$

[1] As usual, we define the lattice spacing such that NNs have unit distance.