where $\langle i, j \rangle$ denotes NN hopping bonds and $\boldsymbol{\tau}_l \equiv \boldsymbol{\tau}_l^A$.

For the hyperdiamond lattice (hd), these hopping vectors can be constructed recursively:

1. start with a regular chain,

$$\boldsymbol{\tau}_1 = (1), \quad \boldsymbol{\tau}_2 = (-1) \quad \text{for } d = 1 \tag{B.3}$$

2. given a set $\{\boldsymbol{\tau}'_l\}$ of primitive lattice vectors in dimension $d-1$, define

$$\boldsymbol{\tau}_l = \begin{cases} (\sqrt{1-\gamma_d^2}\boldsymbol{\tau}'_l, \gamma_d), & l = 1, 2, \ldots, d \\ \underbrace{(0, 0, \ldots, 0, 1)}_{d \text{ components}}, & l = d+1 \end{cases} \tag{B.4}$$

$$\gamma_d = \frac{\gamma_{d-1}}{1-\gamma_{d-1}} = -\frac{1}{d} \,. \tag{B.5}$$

Now, the noninteracting tight-binding Hamiltonian (with NN hopping) in Fourier transformed form remains offdiagonal in the sublattice index $\alpha \in \{A, B\}$,

$$H_0 = \begin{pmatrix} 0 & H_1 \\ H_1^* & 0 \end{pmatrix}, \quad H_1 = -t \sum_{m=1}^{d+1} \mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{\tau}_m}, \tag{B.6}$$

where the eigenvalues form two energy bands $\epsilon_{n\boldsymbol{k}} = \pm|H_1|$. This expression can be simplified by a transformation in momentum space ($\alpha = 1, \ldots, d$),

$$k'_\alpha = \sum_\beta A_{\alpha\beta} k_\beta, \quad A_{\alpha\beta} = (\boldsymbol{a}_\alpha)_\beta, \quad \boldsymbol{a}_\alpha = \boldsymbol{\tau}_{d+1} - \boldsymbol{\tau}_\alpha \,. \tag{B.7}$$

Eliminating an overall phase factor $\mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{\tau}_{d+1}}$ we have

$$\epsilon_{n\boldsymbol{k}} = \pm t\left|1 + \sum_{\alpha=1}^{d} \mathrm{e}^{ik'_\alpha}\right|, \quad -\pi \le k'_\alpha \le \pi, \tag{B.8}$$

which provides a convenient way to compute the DOS and shows that perfect nesting is not possible on the bipartite hd lattice. In the limit $d \to \infty$, where the hopping amplitude has to be scaled as $t = t^*/(d+1)$, the DOS is readily evaluated analytically,

$$\rho^{\mathrm{hd}}(\epsilon) = \frac{|\epsilon|}{t^{*2}} \mathrm{e}^{-\epsilon^2/t^{*2}} \,. \tag{B.9}$$

Calculations of properties which directly depend on $\boldsymbol{k}$ (i.e., not only via $\epsilon_{n\boldsymbol{k}}$), in particular transport properties, however, require knowledge of the transformation matrix $(A_{\alpha\beta})$. As a general example for this problem let us consider the Fermi velocity $\boldsymbol{v}_{\boldsymbol{k}} = -\nabla\epsilon_{\boldsymbol{k}}$ for an energy function defined in terms of transformed k vectors, $\epsilon_{\boldsymbol{k}} = f(\boldsymbol{C}\boldsymbol{k})$ with arbitrary function f and matrix $\boldsymbol{C}$. In the present case, where