the functional form of f is explicitly known, one can first compute the derivative $\boldsymbol{v}'_{\boldsymbol{k}'} = -\nabla_{\boldsymbol{k}'} f(\boldsymbol{k}')$ and then apply the transformation

$$\begin{aligned} \frac{\partial f(\boldsymbol{C}\boldsymbol{k})}{\partial k_\alpha} &= \sum_\beta \frac{\partial f}{\partial k'_\beta} \frac{\partial (\boldsymbol{C}\boldsymbol{k})_\beta}{\partial k_\alpha} = \sum_\beta A_{\beta\alpha} \frac{\partial f}{\partial k'_\beta} && \text{(B.10)} \\ \boldsymbol{v}_{\boldsymbol{k}} &= \boldsymbol{C}^T \boldsymbol{v}'_{\boldsymbol{k}'}\big|_{\boldsymbol{k}'=\boldsymbol{C}\boldsymbol{k}} \,. && \text{(B.11)} \end{aligned}$$

While the transformation matrix defined via (B.3)-(B.5) and (B.7) can be evaluated in any finite dimension, this is in general a complicated task and it is unclear how to take the limit $d \to \infty$ of (B.10). Furthermore, symmetries are hidden so that it is not even apparent if observables which transform like a tensor such as, e.g., the optical conductivity will be isotropic in all dimensions.

Thus, we switch to an alternative representation of the hyperdiamond lattice which embeds the d-dimensional hd lattice into $d+1$ host dimensions in the following way (van Dongen, 1997): starting with a $d+1$-dimensional hypercube with NN hopping we remove every second bond such that the unit vectors connecting to NNs are for sublattices A and B

$$\tilde{\boldsymbol{\tau}}_l^A = \boldsymbol{e}_l, \quad \tilde{\boldsymbol{\tau}}_l^B = -\boldsymbol{e}_l \,. \tag{B.12}$$

Here, $\{\boldsymbol{e}_l\}$ are the usual cartesic unit vectors in $d+1$ dimensions; the tilde ($\sim$) indicates properties in the expanded $d+1$-dimensional vector space. For sake of definiteness we define the origin to belong to the A sublattice. Figure B.1 illustrates this embedding for $d = 2$, i.e., the honeycomb lattice.

It is clear from (B.2) that a tight-binding Hamiltonian defined on a periodic lattice with fixed uniform NN hopping is fully defined by the topology of the hopping bonds and invariant under distortions. Since the lattice defined by (B.12) has $d+1$ NN bonds with a constant angle (here: $\pi/2$) between each pair of them, it (more precisely: each sheet of sites connected by hopping bonds) is indeed topologically equivalent to the usual hyperdiamond lattice and has both the same noninteracting DOS and the same single-particle properties for arbitrary additional local interaction. The noninteracting dispersion is given by

$$\epsilon_{\tilde{\boldsymbol{k}}} = \pm\frac{1}{2}\sqrt{(\epsilon^{hc}_{\tilde{\boldsymbol{k}}})^2 + \tilde{\epsilon}^2_{\tilde{\boldsymbol{k}}}}, \tag{B.13}$$

where $\epsilon^{hc}_{\tilde{\boldsymbol{k}}} = -2t\sum_{l=1}^{d+1}\cos(\tilde{k}_l)$ and $\tilde{\epsilon}_{\tilde{\boldsymbol{k}}} = 2t\sum_{l=1}^{d+1}\sin(\tilde{k}_l)$. Again, one of the $\tilde{k}_l$ could be singled out and be fixed for calculations of the DOS, exposing the equivalence of the dispersions (B.8) and (B.13). Since $\langle(\epsilon^{hc}_{\tilde{\boldsymbol{k}}})^2\rangle = \langle\tilde{\epsilon}^2_{\tilde{\boldsymbol{k}}}\rangle = 2(d+1)t^2$, we can read off from (B.13) that the variance of the DOS of the hyperdiamond lattice is exactly $(d+1)t^2 = t^{*2}$ in any dimension. Note, however, that in the present derivation $\tilde{\boldsymbol{k}}$ remains untransformed within the (halved) Brillouin zone which allows for a direct calculation of the Fermi velocity (in $d+1$ dimensions),

$$(\tilde{\boldsymbol{v}}_{\tilde{\boldsymbol{k}}})_l = \frac{t}{\epsilon_{\tilde{\boldsymbol{k}}}}\left(\epsilon^{hc}_{\tilde{\boldsymbol{k}}}\sin(\tilde{k}_l) + \tilde{\epsilon}_{\tilde{\boldsymbol{k}}}\cos(\tilde{k}_l)\right) \,. \tag{B.14}$$