Appendix D

Linear Response to Electromagnetic Fields

In this appendix, we review the formalism necessary for identifying the quantized Hamiltonian for particles in an electromagnetic field and give a short summary of linear-response theory.

D.1 Electromagnetic Interaction Hamiltonian and Choice of Gauge

Classically, the equation of motion for a nonrelativistic particle with charge q and momentum $m\boldsymbol{v}$ at position $\boldsymbol{r}$ in an external electromagnetic field is[1]

$$m\ddot{\boldsymbol{r}} = q\boldsymbol{E}(\boldsymbol{r},t) + \frac{q}{c}\dot{\boldsymbol{r}} \times \boldsymbol{B}(\boldsymbol{r},t), \tag{D.1}$$

where the right hand side is the Lorentz force. The electric field $\boldsymbol{E}$ and the magnetic field $\boldsymbol{B}$ can be expressed in terms of a vector potential $\boldsymbol{A}$ and a scalar potential ϕ,

$$\boldsymbol{B}(\boldsymbol{r},t) = \nabla \times \boldsymbol{A}(\boldsymbol{r},t); \quad \boldsymbol{E}(\boldsymbol{r},t) = -\nabla\phi(\boldsymbol{r},t) - \frac{1}{c}\frac{\partial \boldsymbol{A}(\boldsymbol{r},t)}{\partial t}. \tag{D.2}$$

A generalization for quantum systems is best derived via the manifestly Lorentz invariant action which for a world line beginning at $a = (ct_1, \boldsymbol{r}_2)$ and ending at $b = (ct_2, \boldsymbol{r}_2)$ reads [see, e.g., Landau and Lifschitz (1992)]

$$S = \int_a^b (-mc^2\,\mathrm{d}\tau - \sum_\alpha \frac{q}{c} A_\alpha\,\mathrm{d}x^\alpha), \tag{D.3}$$

Here, $d\tau = dt/\gamma$, where $\gamma^{-1} = \sqrt{1-(v/c)^2}$; $x^\alpha = (ct, \boldsymbol{r})$ and $A^\alpha = (\phi, \boldsymbol{A})$ are 4-vectors. Rewriting (D.3) in terms of a time integral we can identify the Lagrangian

[1] Here, we use the cgs system. All equations may be transformed into the SI system by multiplying $\boldsymbol{B}$, $\boldsymbol{A}$, and A_α by the speed of light, c.