

# Realistic modeling of strongly correlated materials using LDA+DMFT(QMC): Photoemission spectra of $\text{La}_{1-x}\text{Sr}_x\text{TiO}_3$

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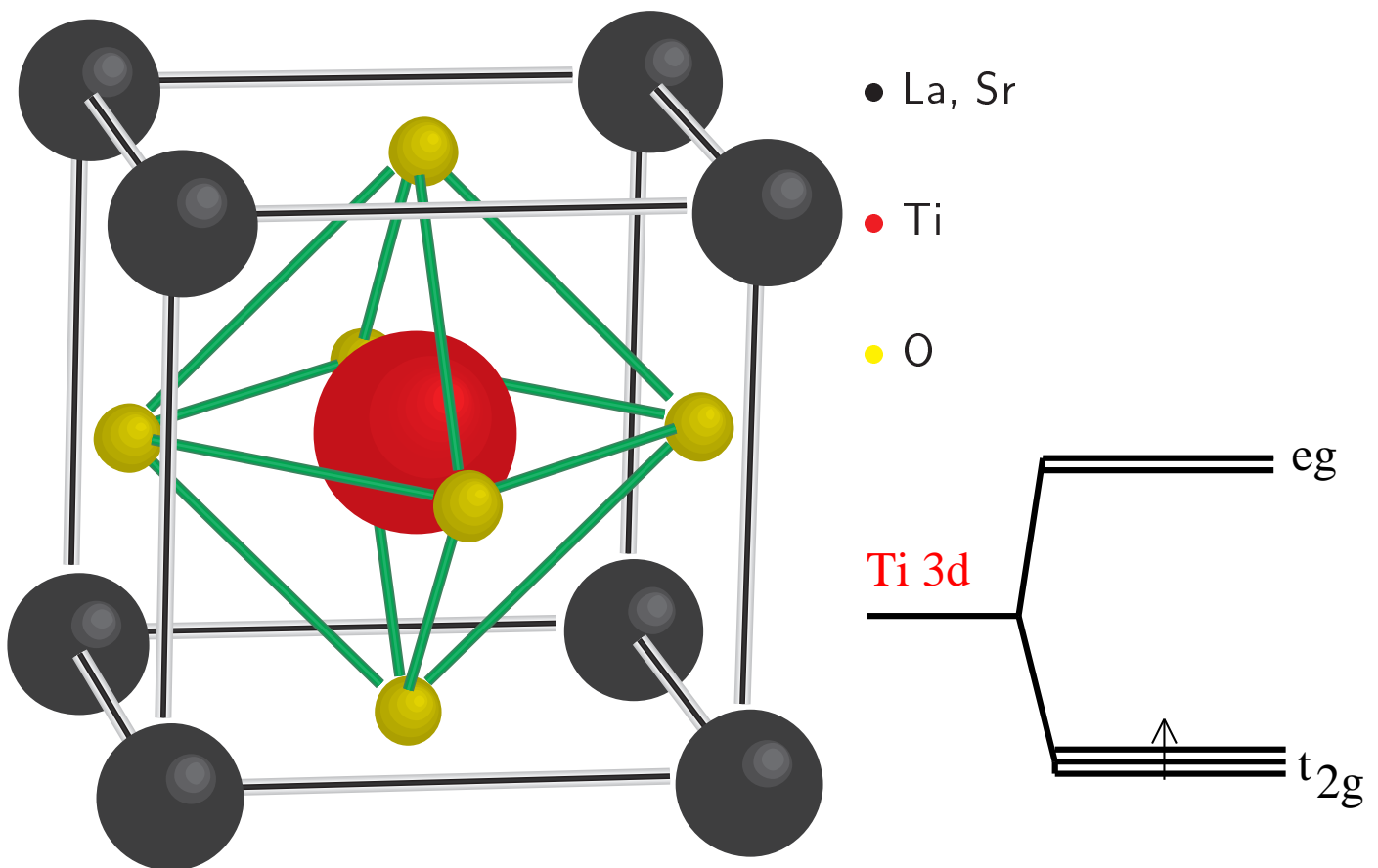
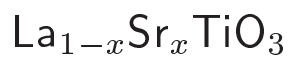
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## Outline

- Introduction: LDA versus model-Hamiltonian approaches the doped Mott insulator  $\text{La}_{1-x}\text{Sr}_x\text{TiO}_3$
- Theory: DFT(LDA); extensions for strong correlations dynamical mean-field theory (DMFT) quantum Monte Carlo (QMC)
- Results: (partial) densities of states photoemission spectra
- Conclusion/Outlook

# Introduction

DFT/LDA	Model Hamiltonians
+ no empirical parameters: "ab initio"	+ systematic many-body investigation possible
+ well-tested code packages	
+ computationally cheap	
- fails for strong correlations	- choice of input parameters?
- self interaction	- computations highly complex
- no systematic improvement	



Cubic Perovskite, slight orthorhombic distortion

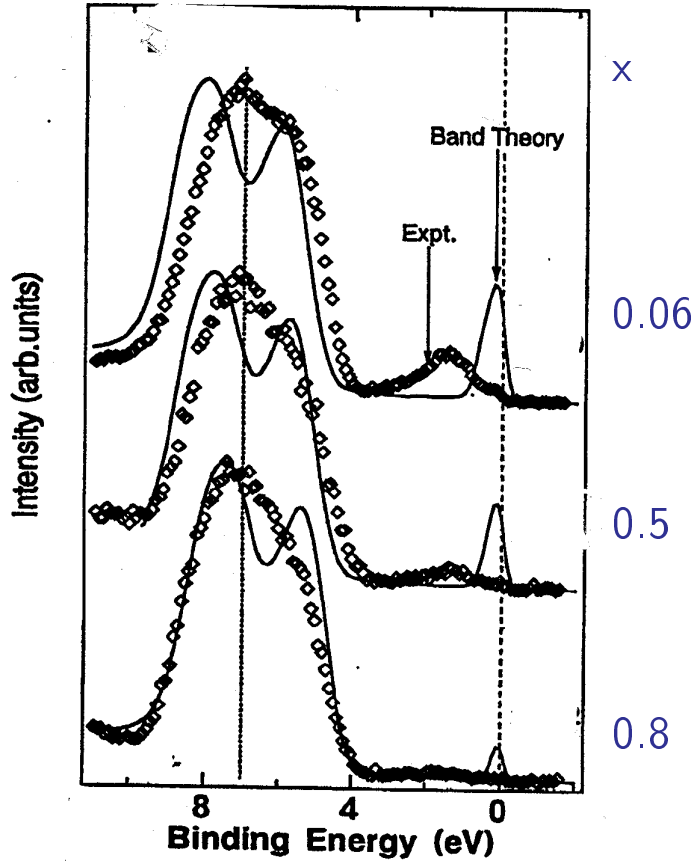
$x = 0$        $\text{LaTiO}_3$ : AF insulator for  $T < T_N = 125\text{K}$   
 $x > 0.05$     strongly correlated paramagnetic metal  
 $x = 1$        $\text{SrTiO}_3$ : band insulator

# Photoemission spectra

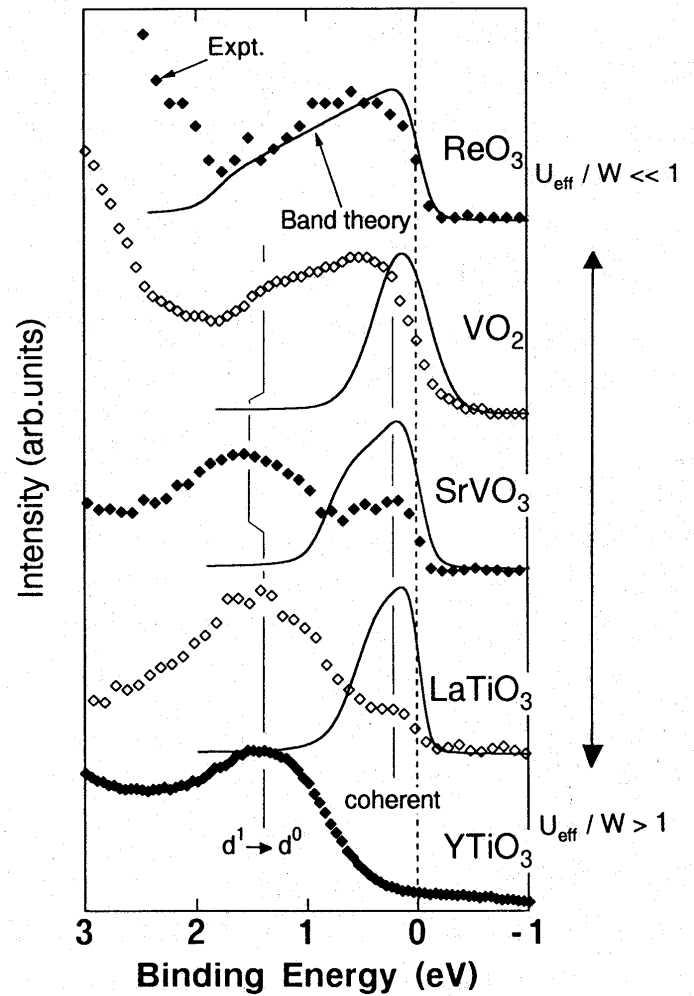


O-2p

Ti-3d



[Fujimori et al., PRB 46, 9841 (1992)]



[Fujimori et al., PRL 69, 1796 (1992)]

- LDA fails when correlations become strong
- $\text{La}_{1-x}\text{Sr}_x\text{TiO}_3$  ( $x \ll 1$ ) is typical strongly correlated transition metal oxide

# Theory

## Density functional theory (DFT):

- mapping onto effective single-particle problem
- ground state method

Kohn-Sham equations:

$$\left[ -\nabla^2 + v_{ext}(\mathbf{r}) + 2 \int d\mathbf{r}' \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} + v_{xc}(\mathbf{r}, \{\rho\}) \right] \varphi_i(\mathbf{r}) = \varepsilon_i \varphi_i(\mathbf{r})$$

## Local (spin) density approximation, L(S)DA:

$$v_{xc}(\mathbf{r}, \{\rho\}) = v_{xc}(\rho_{\uparrow}(\mathbf{r}), \rho_{\downarrow}(\mathbf{r})),$$

obtained from QMC solution of homogeneous electron gas.

- + good for simple metals
  - only Stoner-type long range order
  - gradient corrections
  - self-interaction correction
- } no general improvement

**LDA+U**: introduces Hubbard U for localized orbitals, treatment in static mean-field theory [Anisimov, Zaanen, Andersen, PRB. 44, 943 (1991)]

$$E = E_{LDA} + \frac{1}{2}U \sum_{m\sigma \neq m'\sigma'} n_m^{\sigma} n_{m'}^{\sigma'} - \frac{1}{2}UN(N - 1); \quad N = \sum_{m\sigma} n_m^{\sigma}$$

- + insulating, long-range ordered phases (AF, OO) can be described
- + U can be extracted from constrained supercell LDA
- no Kondo effect, no paramagnetic insulator  
→ dynamical theory needed

# LDA + multi-band Hubbard Hamiltonian

## 1) LDA-LMTO bandstructure calculation → tight-binding model

$$H^{\text{LMTO}} = \sum_{ilm,jl'm',\sigma} \left( t_{ilm,jl'm'} \hat{c}_{ilm,\sigma}^\dagger \hat{c}_{jl'm',\sigma} + \varepsilon_{ilm} \hat{n}_{ilm,\sigma} \delta_{ilm,jl'm'} \right)$$

→  $E^{\text{LMTO}}$ : total LDA energy

## 2) Add local Coulomb correlations

$$H_{\text{corr}} = \frac{1}{2} \sum'_{ilm'l'm'\sigma\sigma'} U_{il,mm'} \hat{n}_{ilm,\sigma} \hat{n}_{il'm',\sigma'} + \text{off-diagonal parts}$$

→  $E_{\text{corr}}^{\text{LMTO}}(\{n_{d,f}\})$ : average correlation energy

only single  $d$ -orbital:  $E_{\text{corr}}^{\text{LMTO}} = \frac{1}{2} U n_d (n_d - 1)$

## 3) Subtract local Coulomb correlations included in LDA

shift localized levels

$$\varepsilon_{ilm} \longrightarrow \varepsilon_{ilm}^0 = \begin{cases} \varepsilon_{ilm}, & l \neq l_d \\ \varepsilon_{ilm} - U(n_d - \frac{1}{2}), & l = l_d \end{cases}$$

$$H_0^{\text{LMTO}} := H^{\text{LMTO}} \{t_{ilm,jl'm'}, \varepsilon_{ilm}^0\} \quad \text{“non-interacting part”}$$

$$H = H_0^{\text{LMTO}} + H_{\text{corr}} \quad \text{“ab initio Hamiltonian”}$$

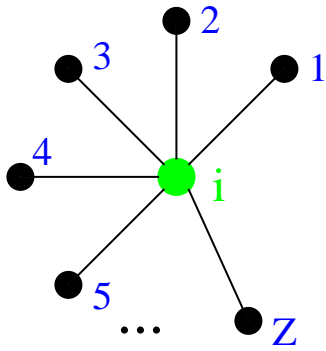
Approximation needed for treatment of  $H_{\text{corr}}$ :

DMFT: Anisimov, Poteryaev, Korotin, Anokhin, Kotliar, J. Phys. **9**, 7359 (1997)

FLEX: Lichtenstein, Katsnelson, PRB **57**, 6884 (1998)(also Hubbard-I, DMFT)

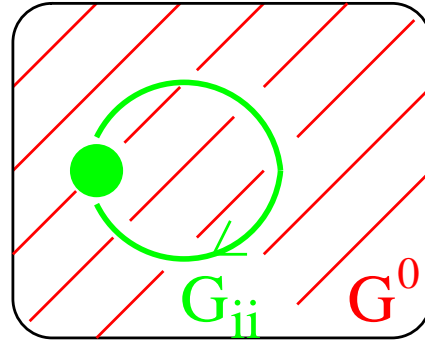
Drchal, Janiš, Kudrnovský, PRB **60**, 15664 (1999)

# LDA + DMFT



lattice problem

DMFT  
 $\xrightarrow{\text{exact for } Z \rightarrow \infty}$



single impurity Anderson model  
 + self consistency condition

Metzner, Vollhardt, PRL **62**, 324 (1989); Georges et al., RMP **44**, 187 (1995)

(i) self-consistency condition ( $\mathbf{k}$ -integrated Dyson equation)

$$G_{qlm,q'l'm'}^\sigma(z) = \frac{1}{v_{\text{BZ}}} \int d\mathbf{k} \left[ z - H_{0,qlm,q'l'm'}^{\text{LMTO}}(\mathbf{k}) - \Sigma_{qlm,q'l'm'}^\sigma(z) \delta_{ql,q_d l_d} \right]^{-1}$$

$q, q'$ : atomic index in unit cell

$\uparrow$  local selfenergy

For weak hybridization of  $d$ -orbital, orbital/spin-degenerate case:

$$G \equiv G|_d, \Sigma \text{ diagonal} \longrightarrow G(z), \Sigma(z) \quad \text{for } l = l_d, q = q_d$$

$$G(z) = G^0(z - \Sigma(z)) = \int d\varepsilon \frac{\rho^0(\varepsilon)}{z - \Sigma(z) - \varepsilon}$$

(ii) multi-band SIAM

$$G_{\nu m \sigma} = -\langle \psi_{\nu m \sigma} \psi_{\nu m \sigma}^* \rangle_{\mathcal{A}\{\psi, \psi^*; G^{-1+\Sigma}\}}; \quad z \rightarrow \omega_\nu = \frac{(2\nu + 1)\pi}{\beta}$$

Numerical solution of LDA+DMFT(X)

- IPT      Anisimov, Poteryaev, Korotin, Anokhin, Kotliar, J. Phys. **9**, 7359 (1997)
- NCA      Zöfl, Pruschke, Keller, Poteryaev, Nekrasov, Anisimov, PRB **61**, 12810 (2000)
- QMC      Katsnelson, Lichtenstein, PRB **61**, 8906 (2000)
- Liebsch, Lichtenstein, PRL **84**, 1591 (2000)

QMC solution of impurity problem ( $M$  interacting orbitals):

- discretization  $\Delta\tau$  of imaginary time  $\beta = \Lambda\Delta\tau$
- Trotter decomposition
- discrete Hubbard-Stratonovich transformation  
→  $\Lambda$ -component Ising field for each pair of interacting orbitals/spins  
 $\hat{=} 2^{M(2M-1)\Lambda}$  different non-interacting systems
- MC importance sampling over auxiliary Ising fields
- total computational cost  $\propto M(2M - 1)\beta^3$

Hirsch and Fye, PRL **56**, 2521 (1986)

M. Jarrell, PRL **69**, 168 (1992); George et al., RMP **44**, 187 (1995)

Rozenberg, PRB **55**, R4855 (1997); Held, Vollhardt, EPJ B **5**, 473 (1998)

Analytic continuation onto real axis using Maximum Entropy method

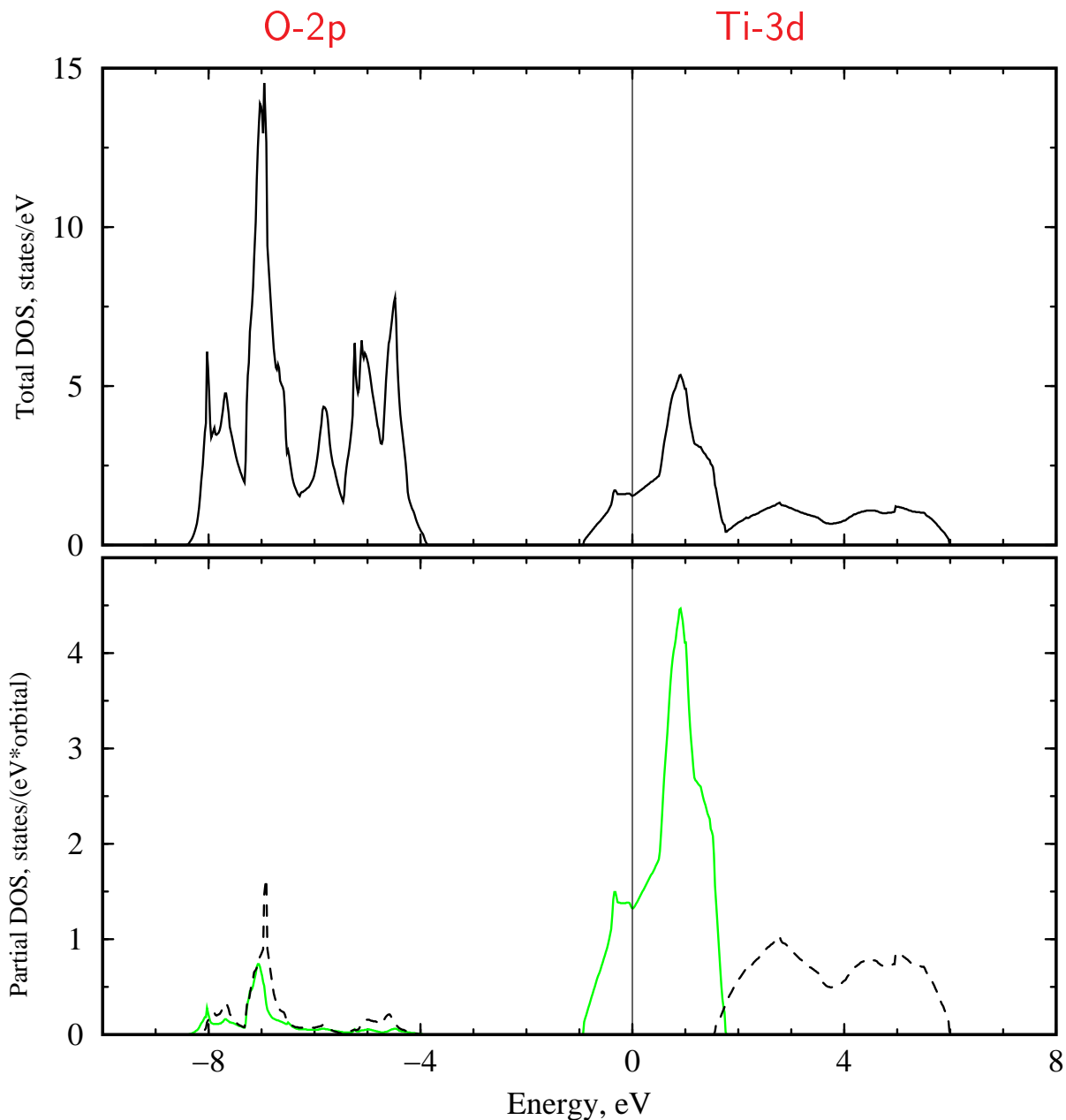
- ill-conditioned inversion of  $G(\tau) = \int d\omega \rho(\omega) e^{-\tau\omega} / (1 + e^{-\beta\omega})$
- minimize  $\chi^2$  and entropy function
- can be biased by choice of default model

Jarrell, Gubernatis, Physics Reports **269**, 133 (1996)

# Results

## LDA-LMTO calculation for $\text{LaTiO}_3$

Nekrasov, Held, Blümer, Poteryaev, Anisimov, Vollhardt  
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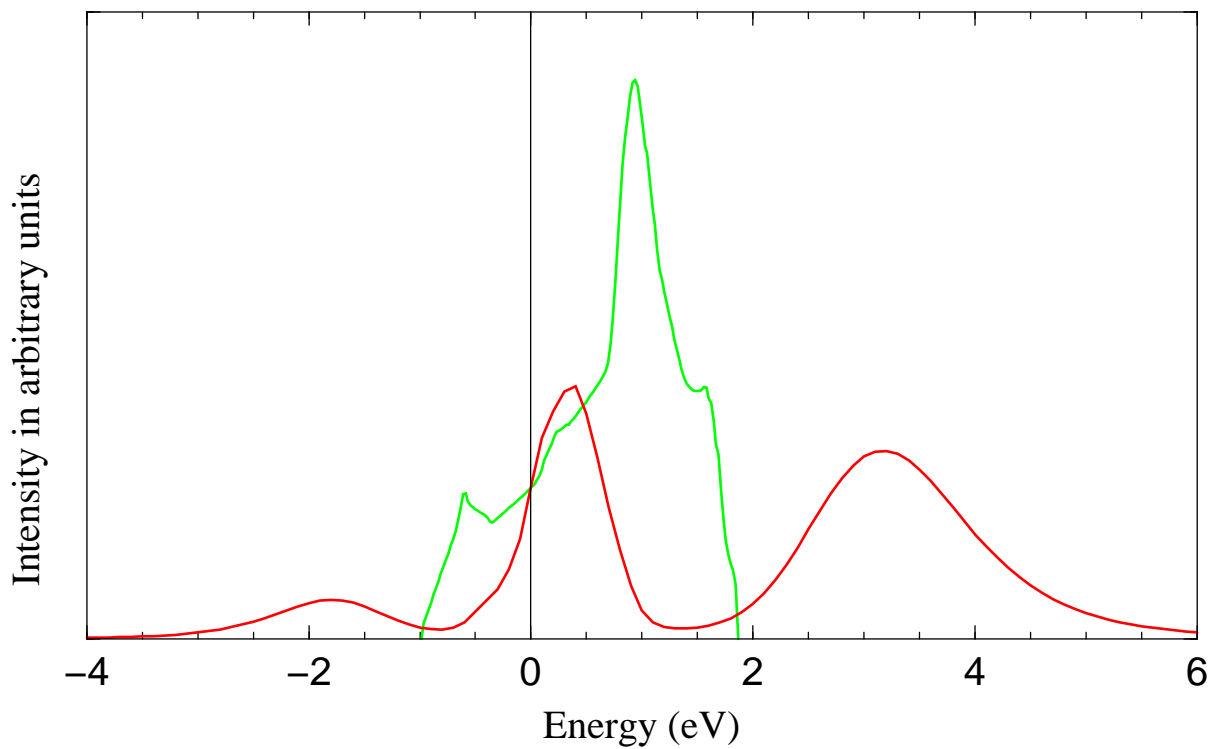
Upper figure: total density of states;  
lower figure: partial  $t_{2g}$  (—) and  $e_g$  (- -) densities of states.

Orthorhombic structure approximated by cubic structure with same volume  
—→ slight overestimation of effective bandwidth

# DMFT calculation for $\text{La}_{1-x}\text{Sr}_x\text{TiO}_3$

Nekrasov, Held, Blümer, Poteryaev, Anisimov, Vollhardt  
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Partial  $t_{2g}$  densities of states



LDA (—)

LDA+DMFT(QMC) (—)

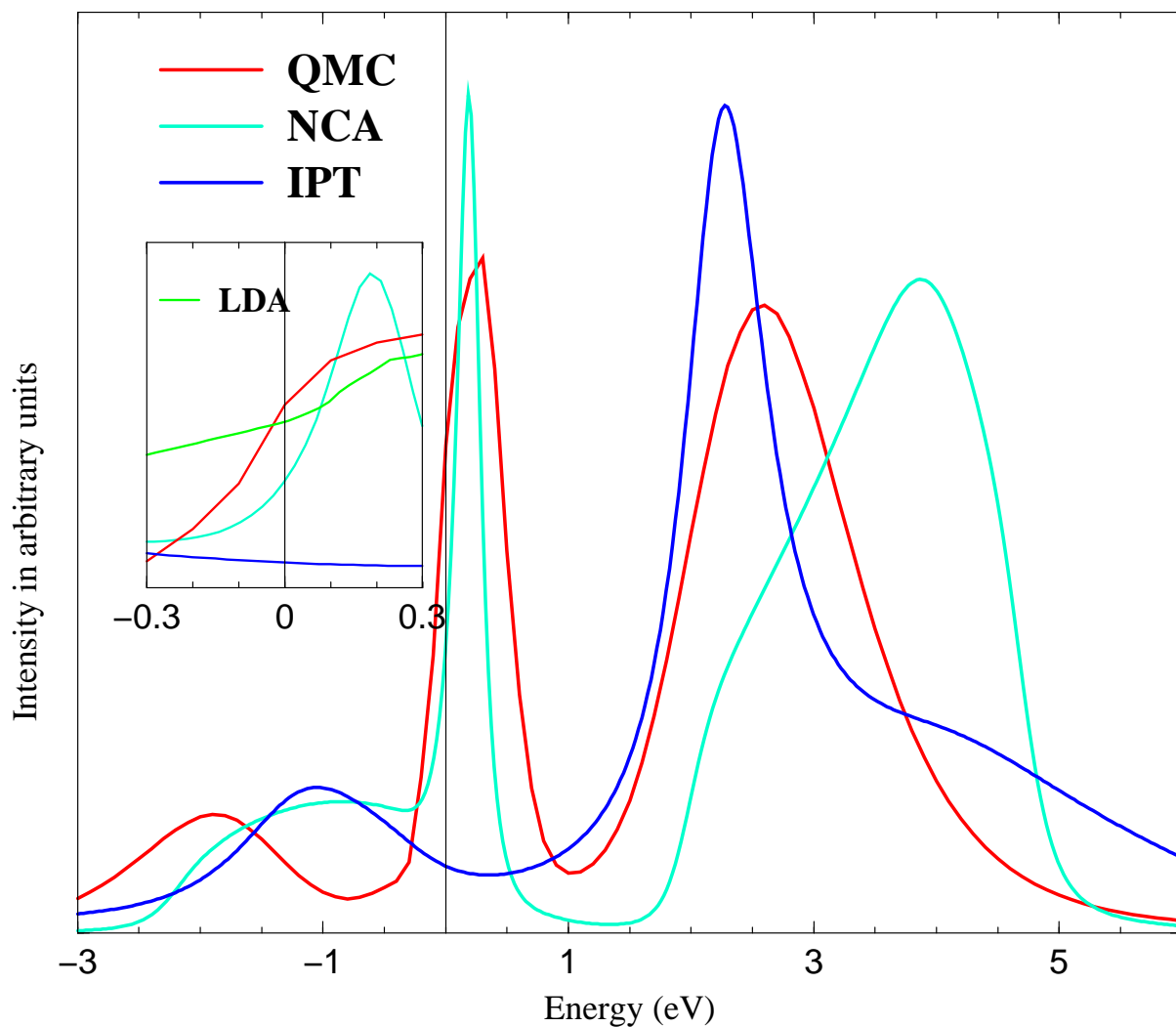
Parameters for DMFT:

$x = 0.06$ ,  $U = 4.0$  eV,  $T = 0.1$  eV  $\sim$  1000K, 3 degenerate bands

# Comparison of different DMFT methods

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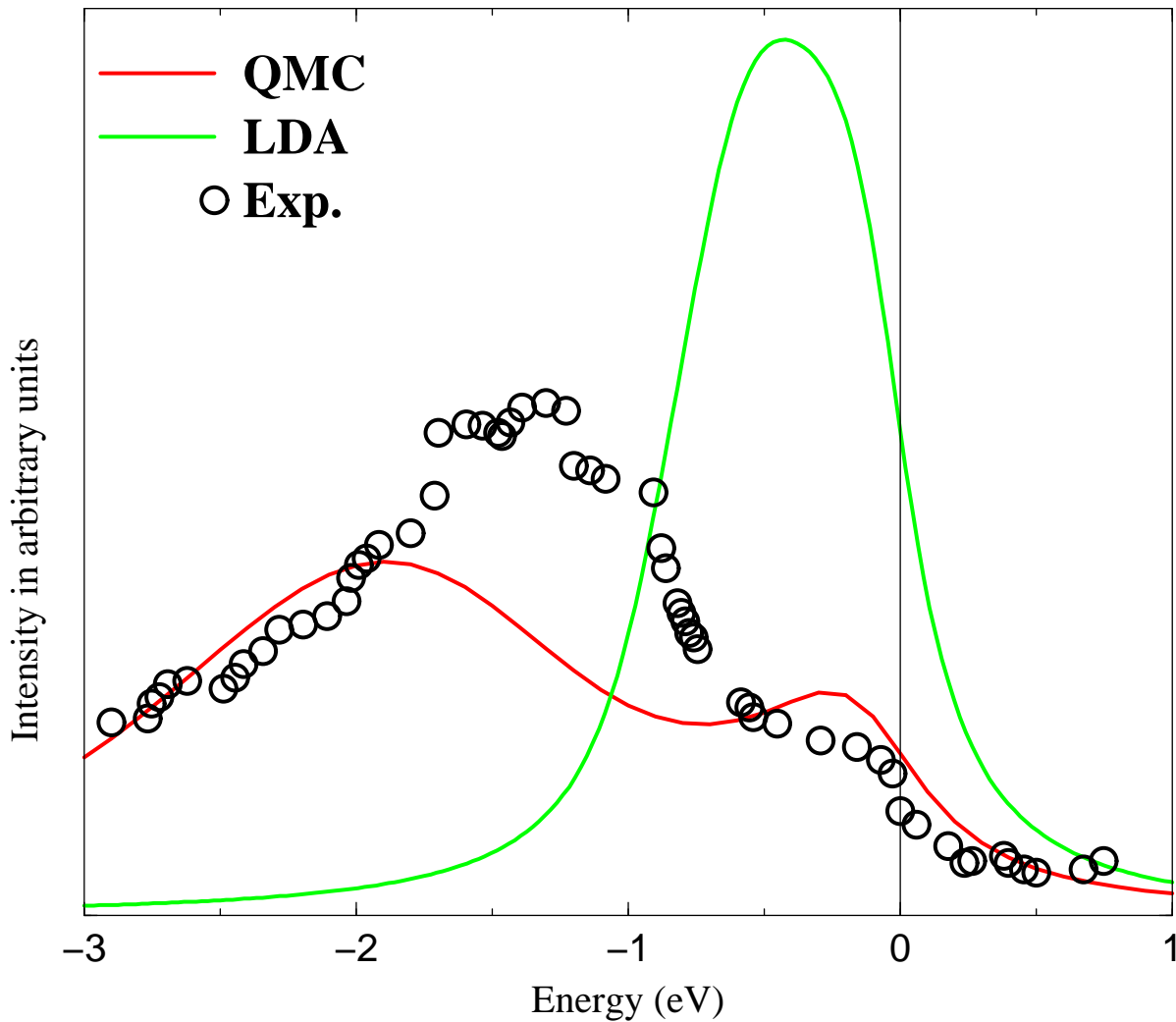
Spectral densities of  $\text{La}_{1-x}\text{Sr}_x\text{TiO}_3$  as calculated by LDA+DMFT



QMC and approximations IPT and NCA at  $x = 0.06$ ,  $T = 0.1$  eV.  
Inset: Behavior at Fermi level including LDA DOS ( $x = 0$ ,  $T = 0$ ).

# Comparison with experiment

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Comparison of the experimental photoemission spectrum ( $T = 80$  K, resolution  $\Delta\varepsilon = 0.3$  eV) [Fujimori et al., PRL **69**, 1796 (1992)], the LDA result ( $T = 0$ ), and the LDA+DMFT(QMC) calculation ( $T = 1000$  K) for  $\text{LaTiO}_3$  with 6% hole doping.

# Conclusion

LDA + DMFT(QMC) reproduces

- qualitative picture of spectral weight transfer from qp-band to lower (and upper) Hubbard band
- position of lower Hubbard band
- relative intensities between qp-band and lower Hubbard band
- narrowing of qp-band

big improvement over LDA

substantial deviations from NCA, IPT

# Outlook

- check influence of Hund's rule couplings
- use  $\mathbf{k}$ -dependent self-consistency equation
- other materials
- full self-consistency: QMC output  $\longrightarrow$  LDA

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