

# Orbital-selective Mott transitions

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Krunoslav Požgajčić, Univ. Frankfurt

## Outline

Introduction: **orbital**/**flavor**-selective Mott transitions in  
**correlated materials**/**ultracold atoms on optical lattices**

Orbital-selective Mott transitions in 2-band Hubbard model

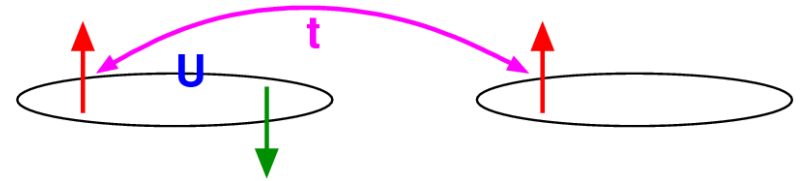
Order of wide-band transition at  $T > 0$  and  $T = 0$

## Summary

# Introduction

Reminder: Mott transition in frustrated 1-band Hubbard model

$$H = -t \sum_{\langle i,j \rangle, \sigma} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

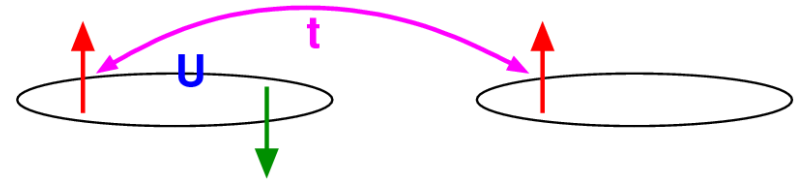


Fundamental question: **smooth crossover** from metal to insulator or **transition at  $U_c(T)$**  (within DMFT)?

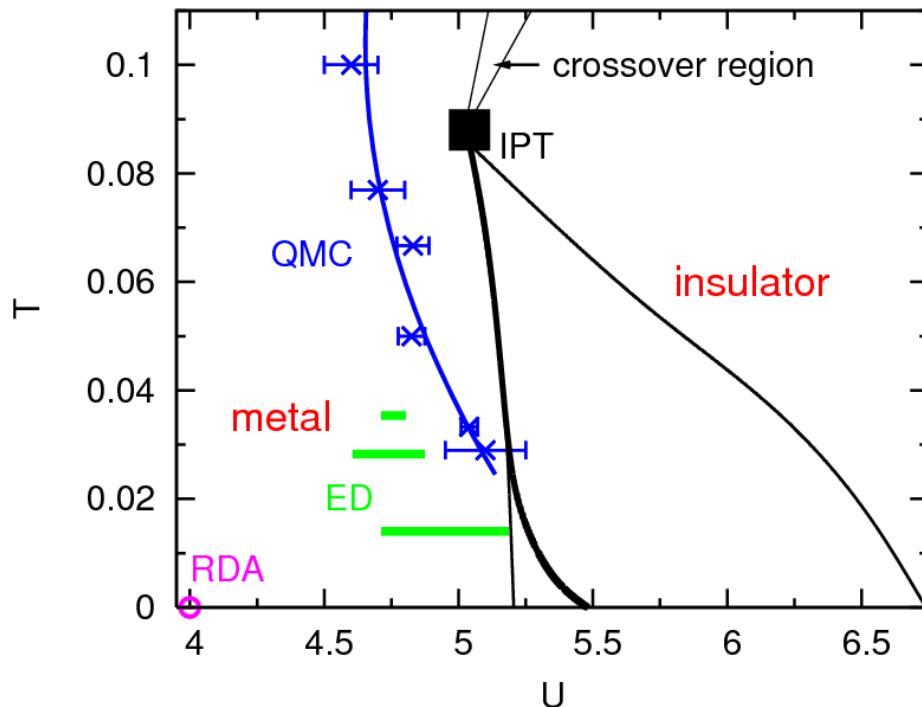
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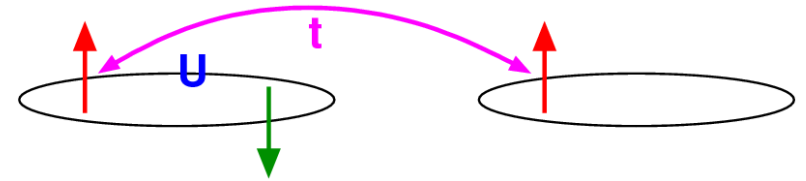


No conclusive answer up to 1999

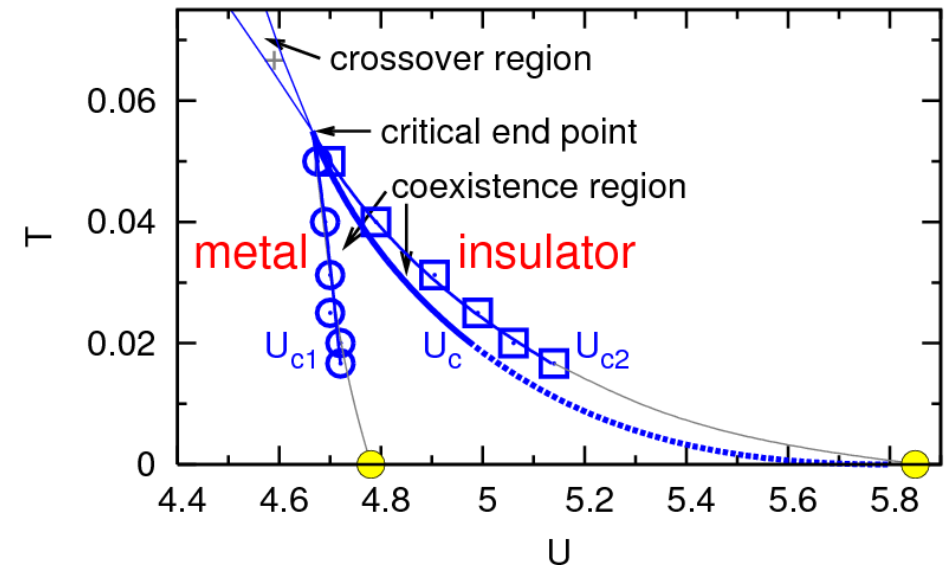
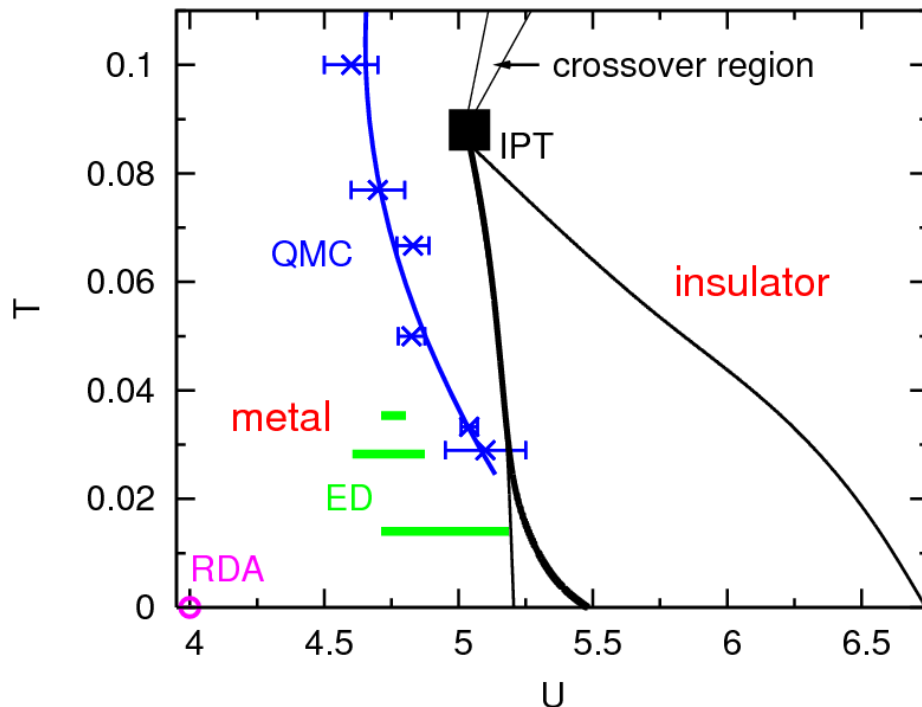
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Now accepted: **1<sup>st</sup> order transition** for  $0 < T < T^*$  (QMC + ePT, 2000-2005)

## Multiorbital case ( $M > 1$ orbitals, half filling $n = M$ )?

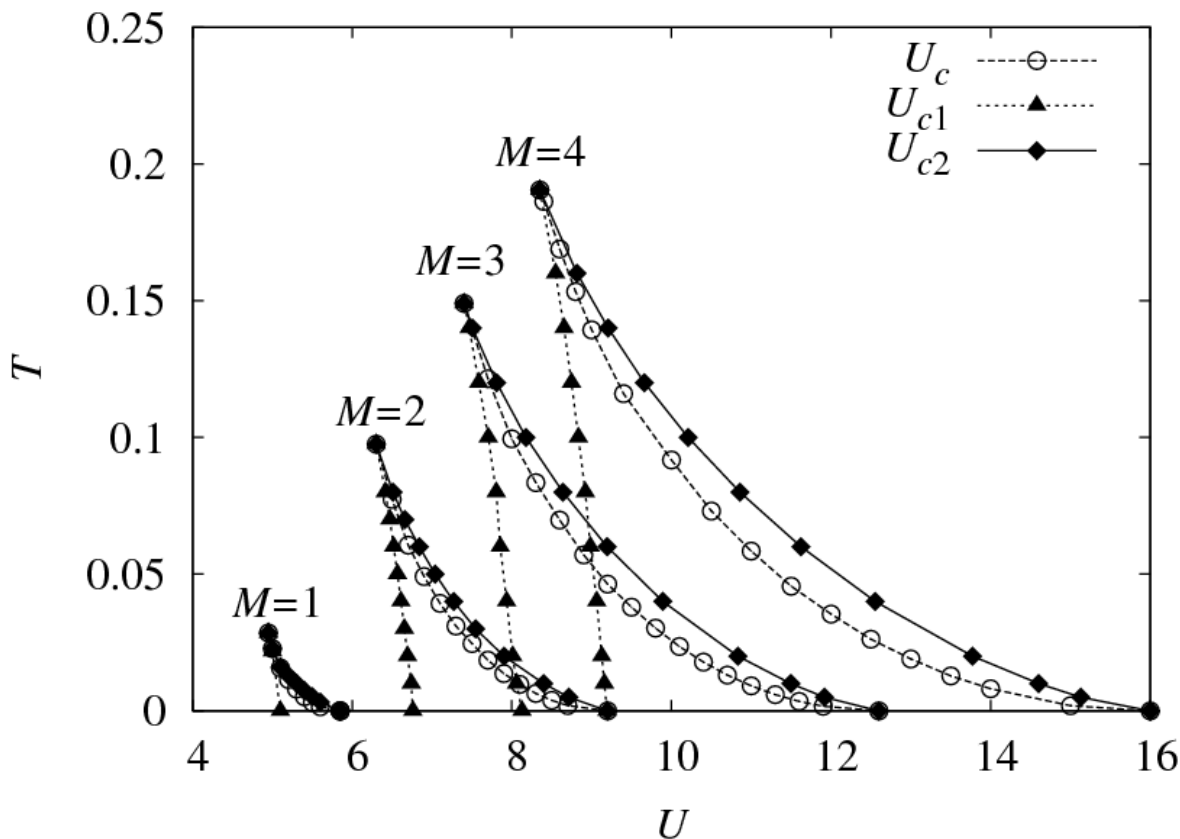
Extension without additional parameters:  $SU(2M)$  symmetric case:

$$H = -t \sum_{\langle i,j \rangle, m, \sigma} c_{im\sigma}^\dagger c_{jm\sigma} + \frac{U}{2} \sum_i \sum_{(m,\sigma) \neq (m'\sigma')} n_{im\sigma} n_{im\sigma'}$$

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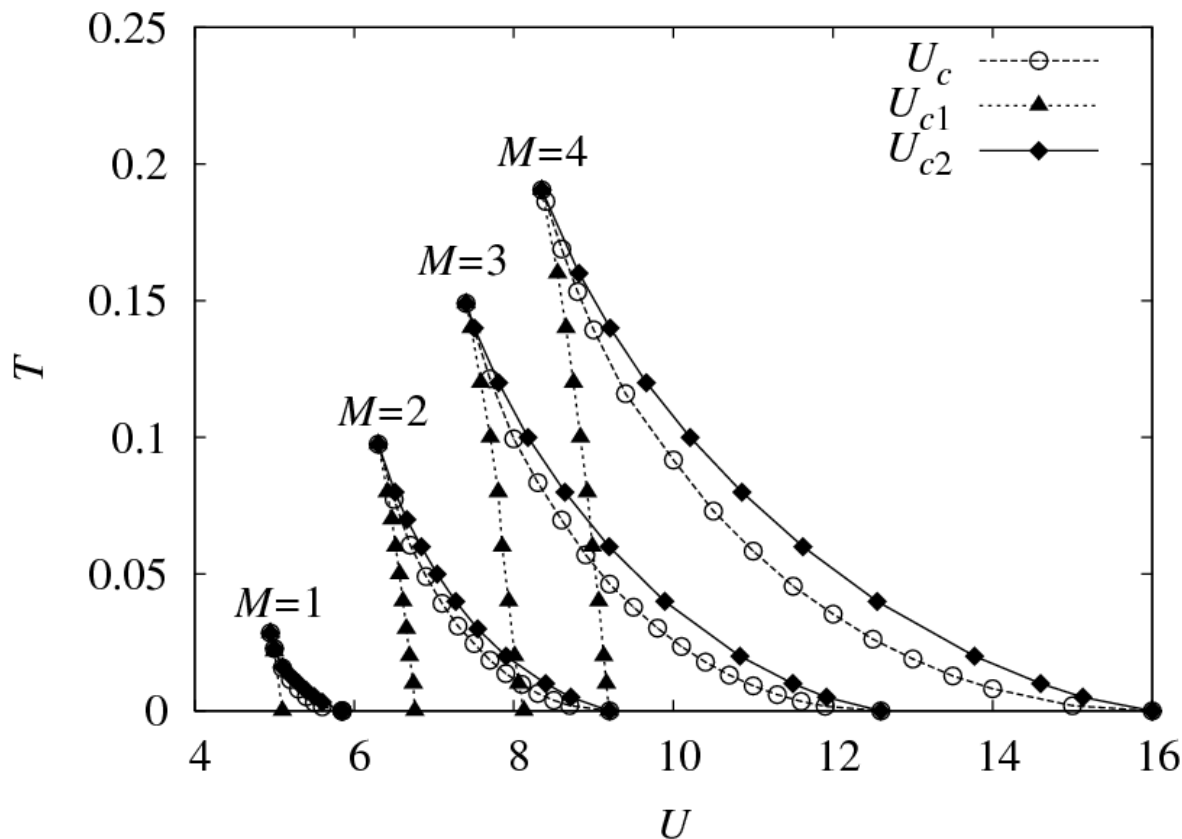


[Inaba, Koga, Suga, Kawakami, PRB 72, 085112 (2005)]

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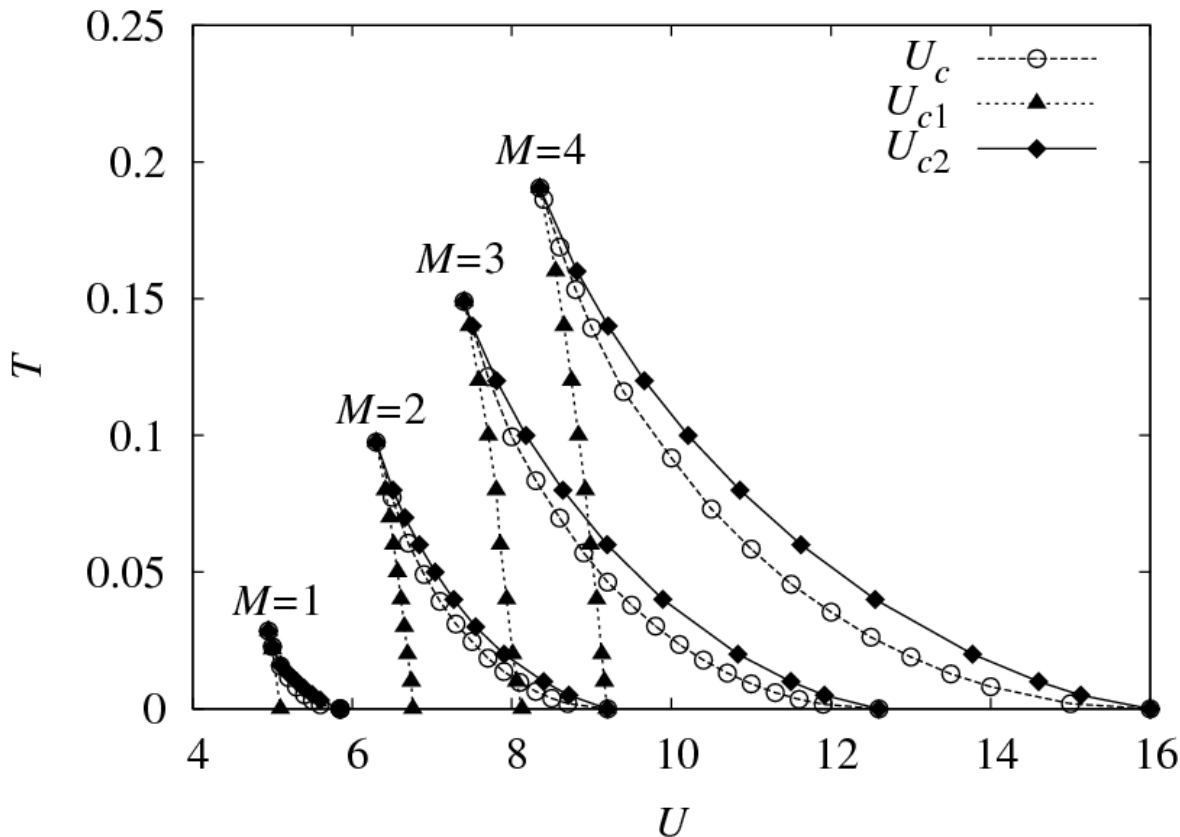
$U_c(M) \propto \sqrt{M}$   
[Koch et al., PRB(1999)]

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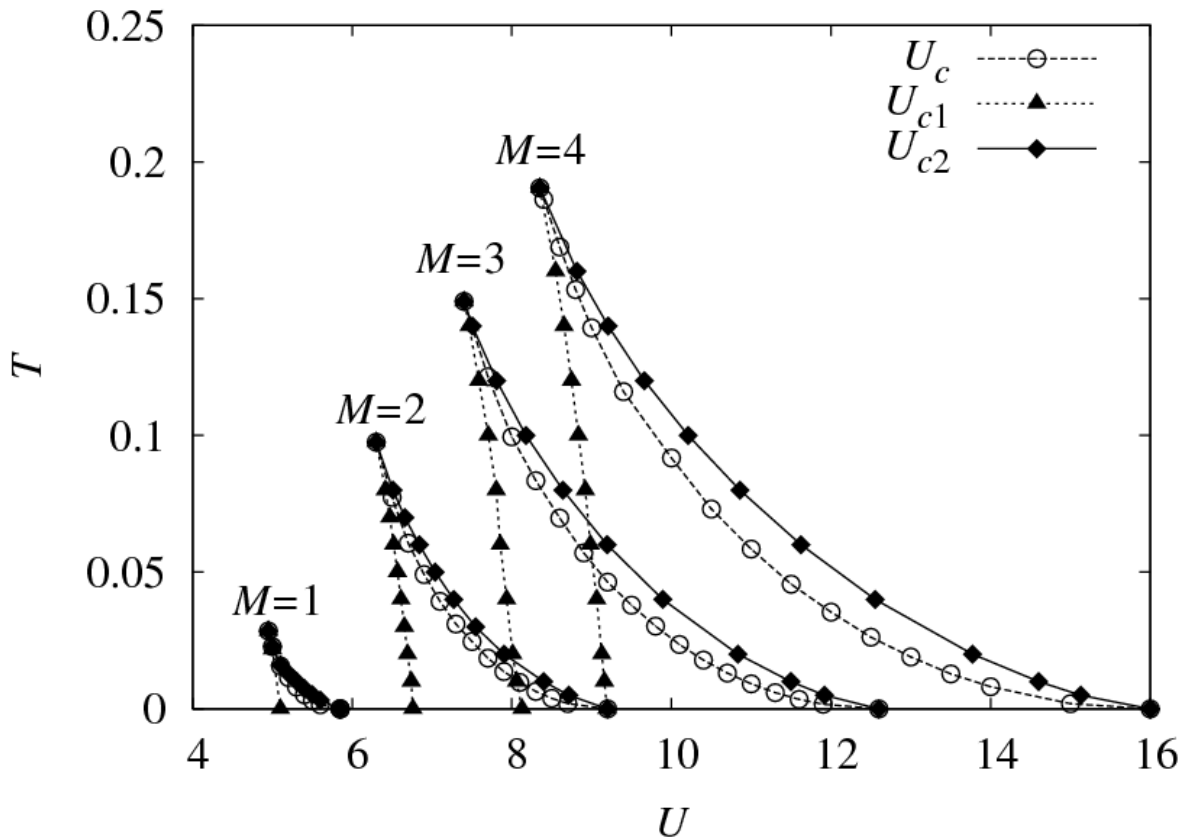
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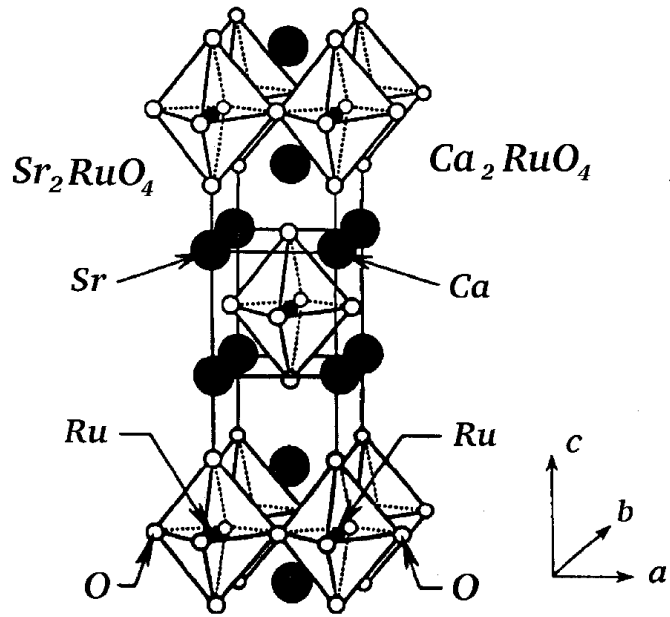
Inclusion of  $J \rightsquigarrow 1^{\text{st}}$  order

[Bünemann, Weber, PRB (1997),  
Ono, Potthoff, Bulla, PRB (2003)]

but still **single Mott transition**

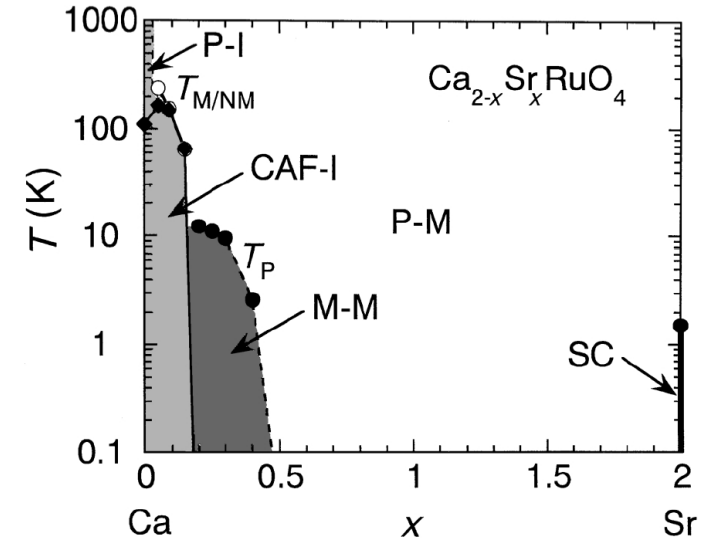
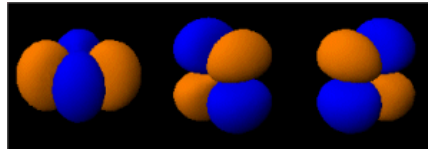
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# OSMTs in $\text{Ca}_{2-x}\text{Sr}_x\text{RuO}_4$



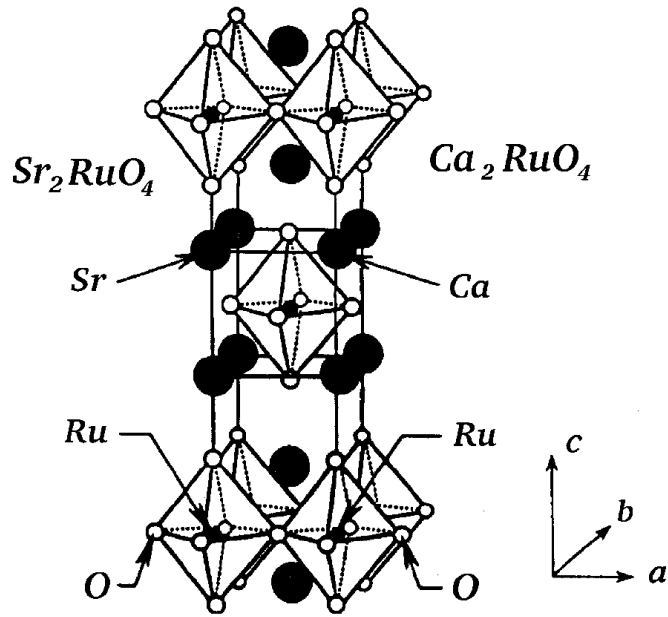
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 $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$

4 valence electrons  
in 3 Ru  $t_{2g}$  orbitals



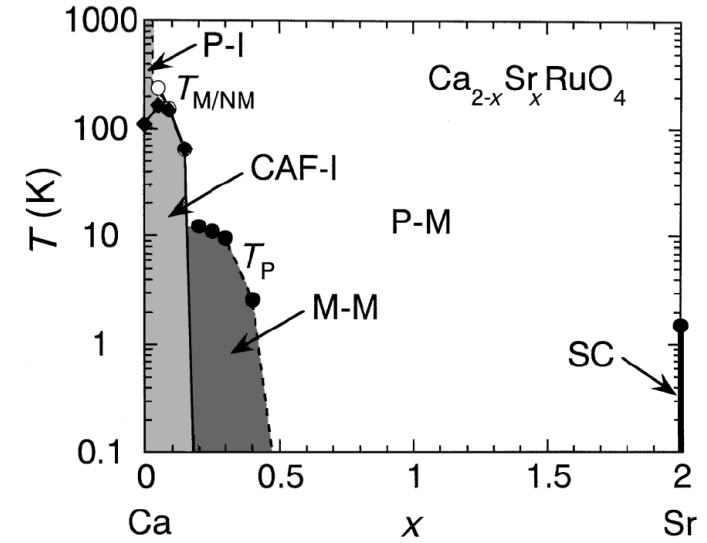
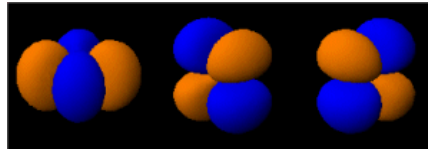
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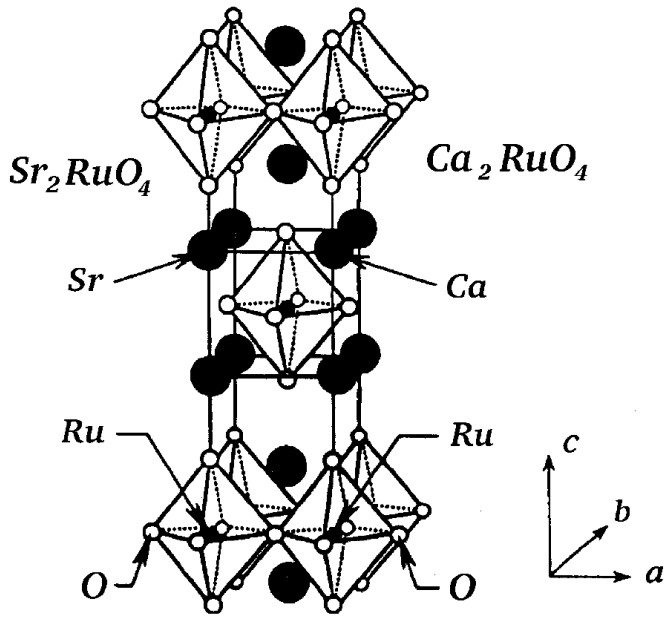


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susceptibility, MR  $\rightsquigarrow$   $S = 1/2$  system (+ easy axis) for  $0.2 < x \lesssim 0.5$  (not  $S = 1$ )

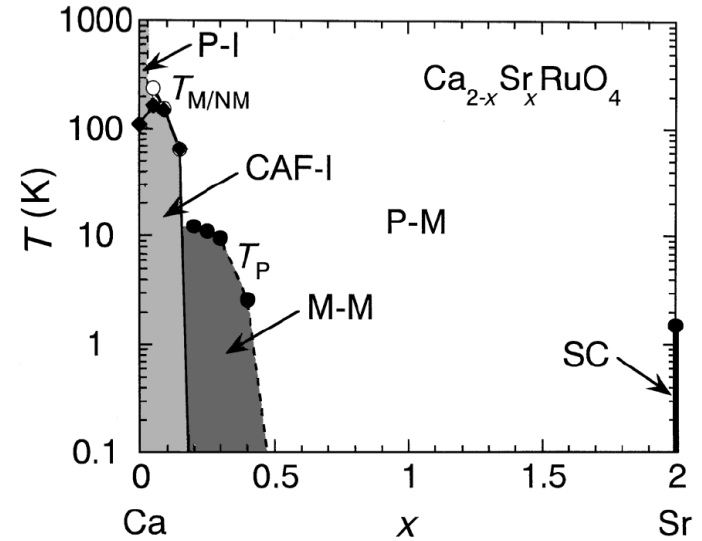
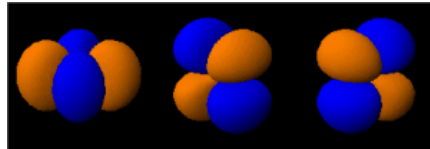
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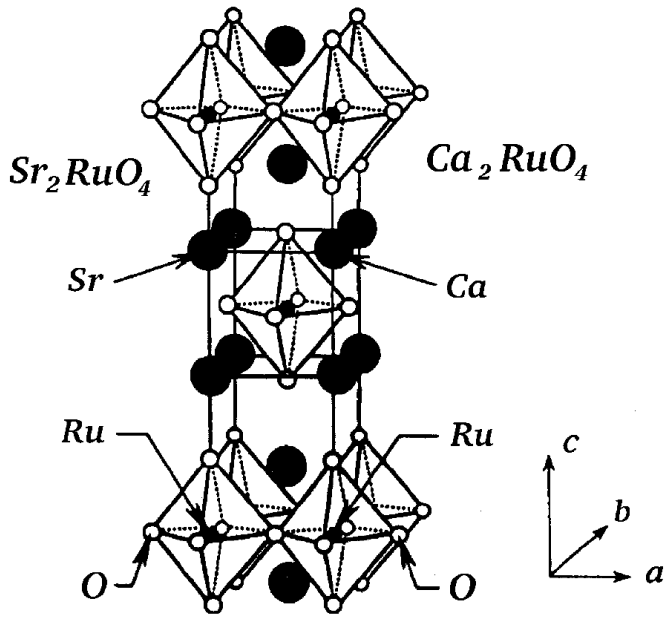
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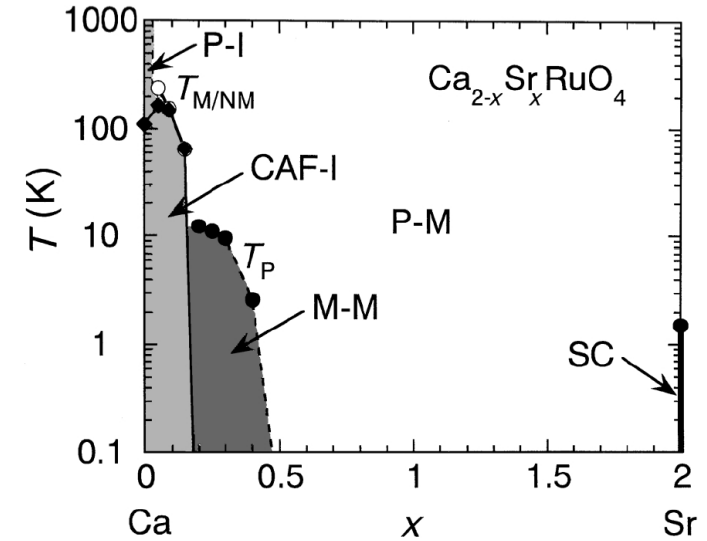
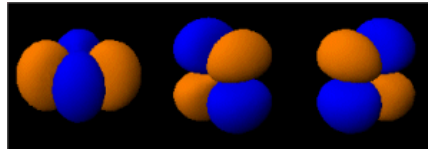
Possible mechanism: different effective hopping amplitudes for in-plane and out-of-plane orbitals [Anisimov et al., EPJB (2002)]

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Related: Dual nature of 5f electrons in  $\text{UPt}_3$  [Zwicknagl, Yaresko, Fulde, PRB (2002)]

# Ultracold fermionic atoms on optical lattices

Electronic case ( $B = 0$ ): exact 2-fold spin degeneracy

in cubic system: orbital degeneracy ( $2 \times e_g, 3 \times t_{2g}, \dots$ )

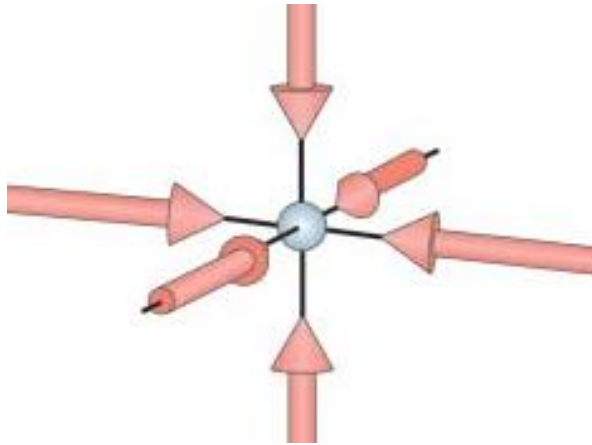
neglected: core states, long-range Coulomb, phonons . . .

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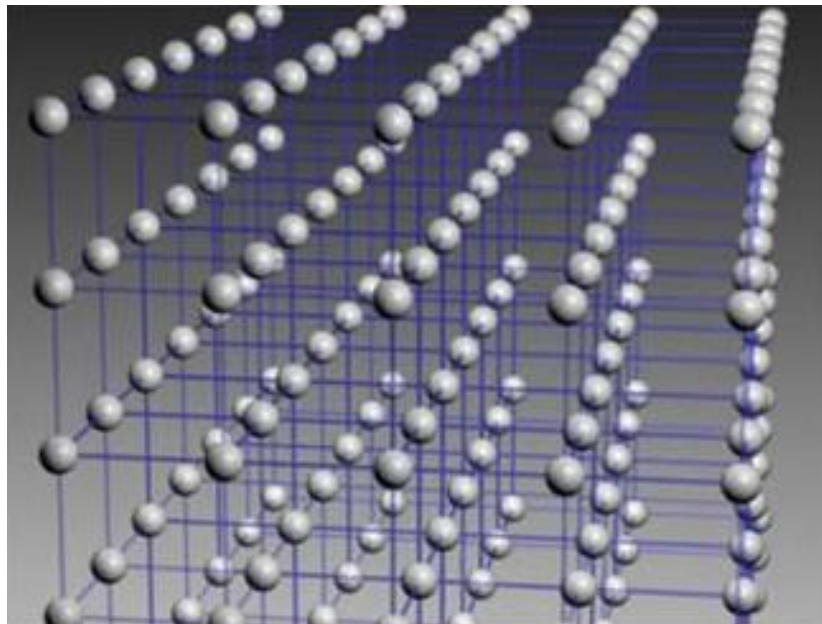
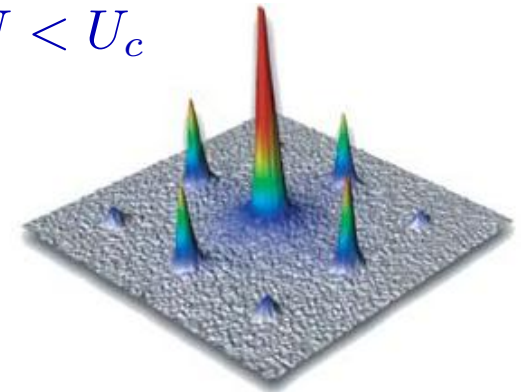
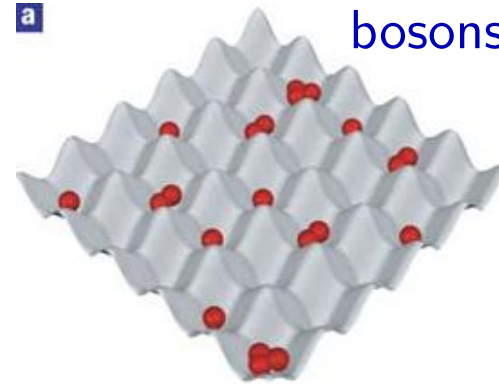
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**a**



Alternative quantum systems with strong correlations: ultracold atoms on optical lattices

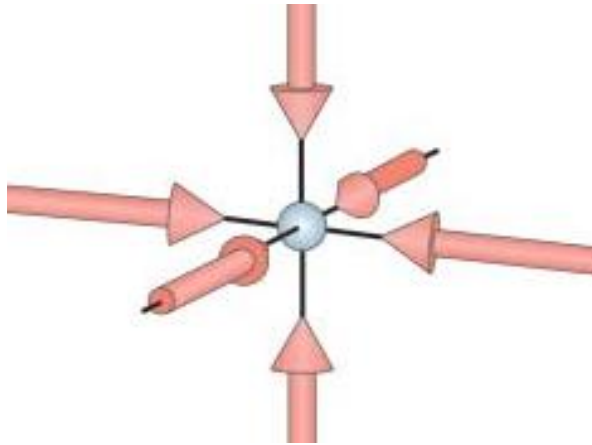
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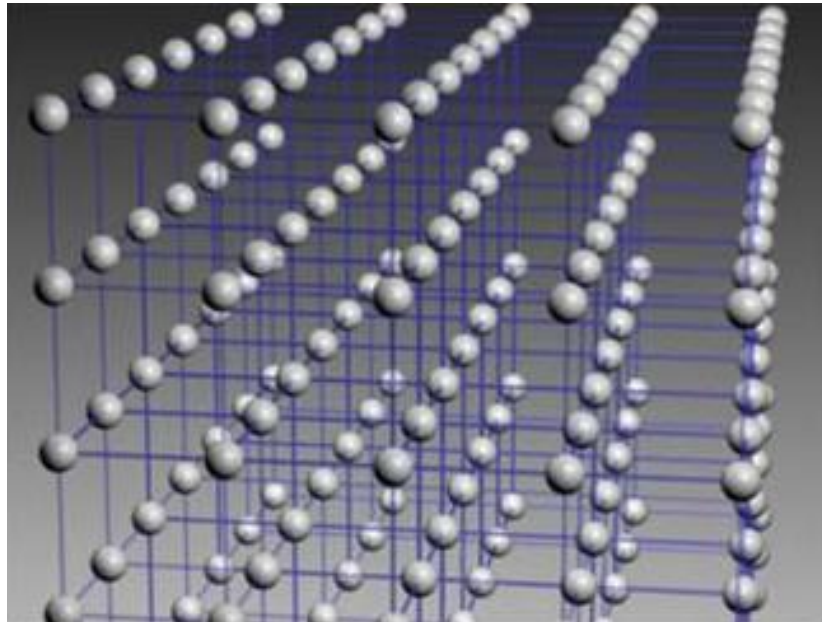
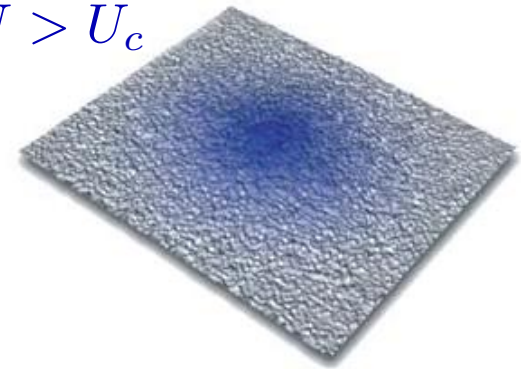
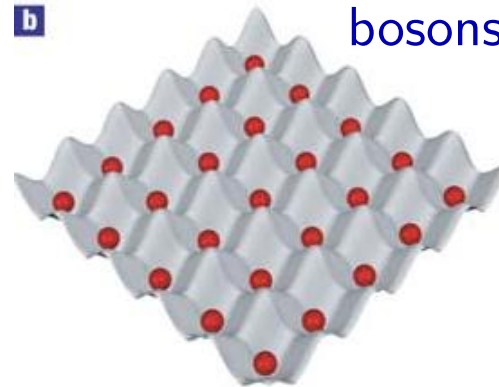
in cubic system: orbital degeneracy ( $2 \times e_g, 3 \times t_{2g}, \dots$ )

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**b**

bosons,  $U > U_c$



Alternative quantum systems with strong correlations: ultracold atoms on optical lattices

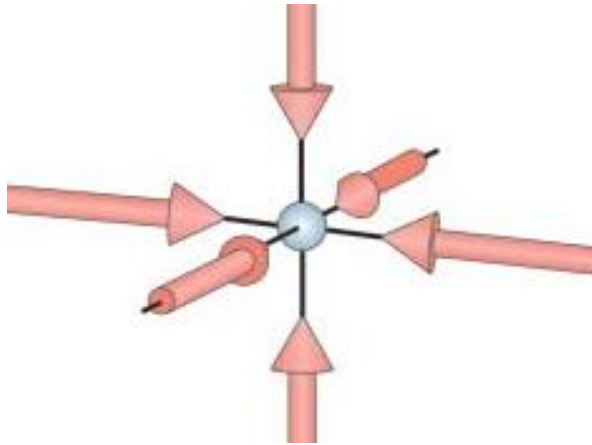
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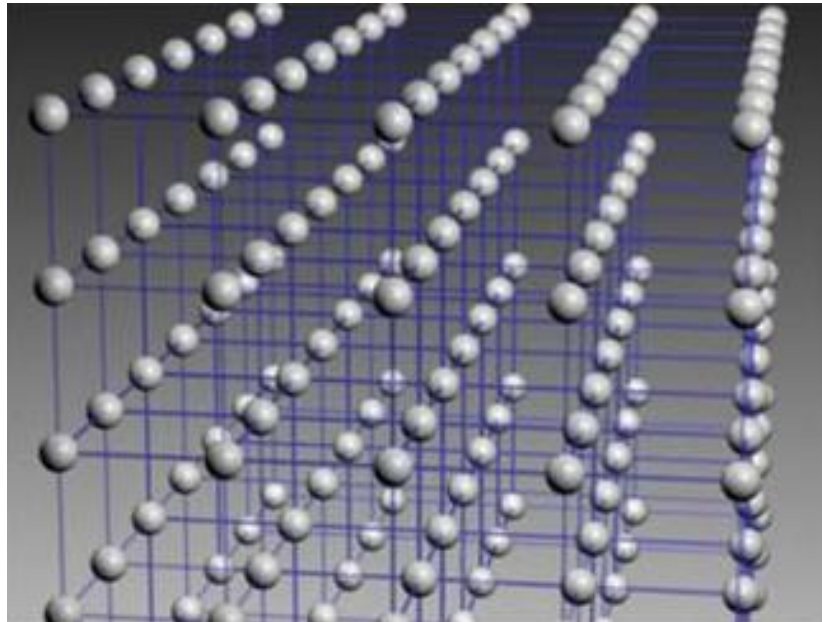
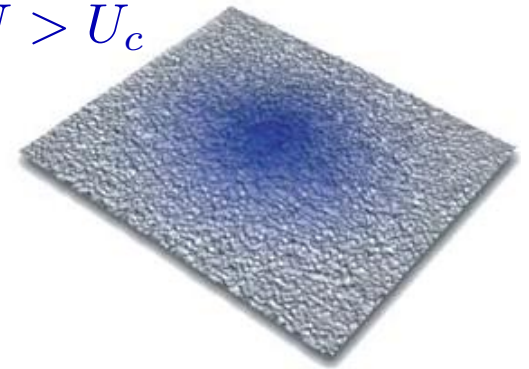
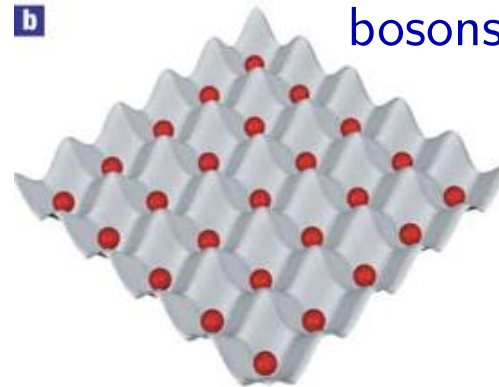
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**Flavors:** species:  ${}^6\text{Li}$ ,  ${}^{40}\text{K}$

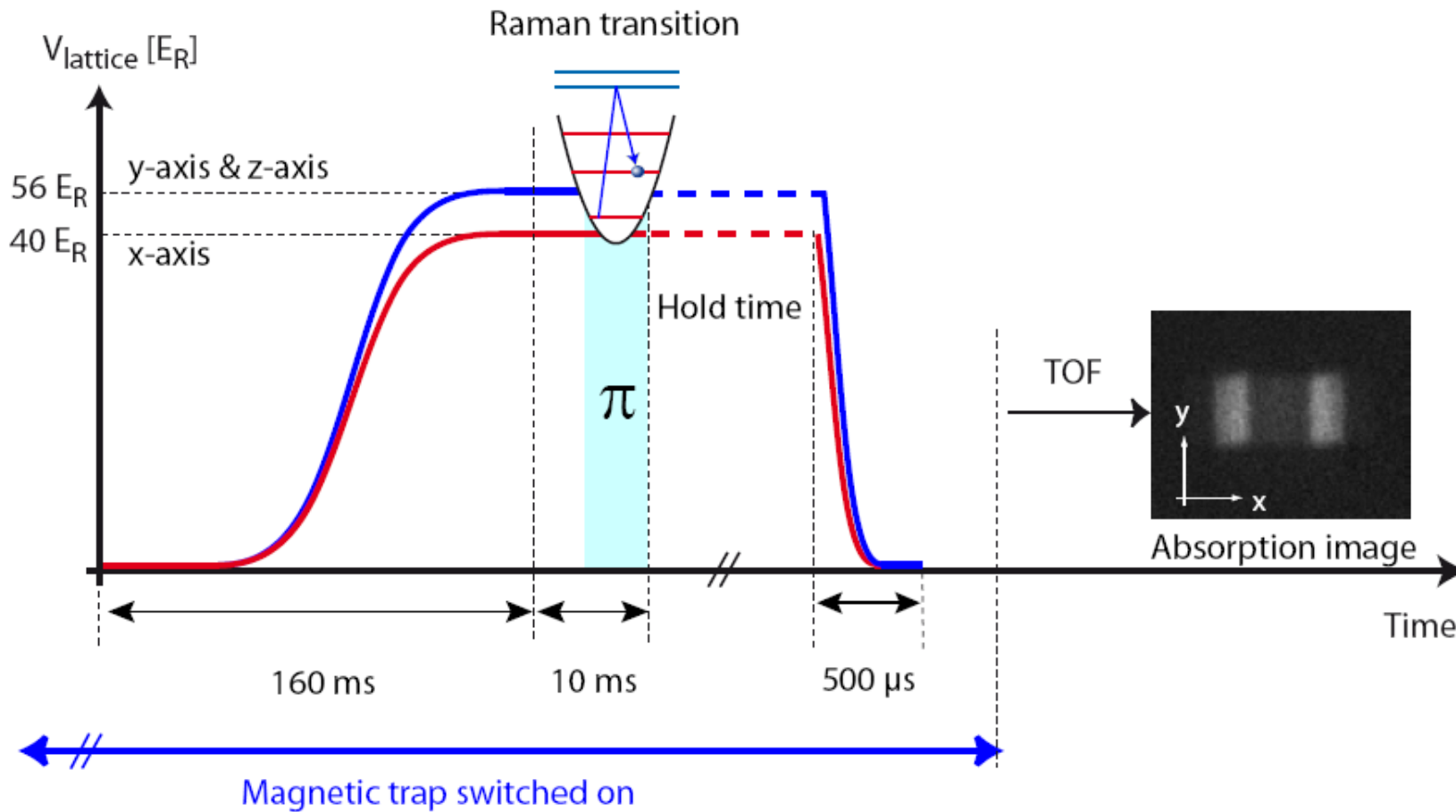
hyperfine states (spin and projections)

vibrational levels ( $\sim$  Raman pulses)

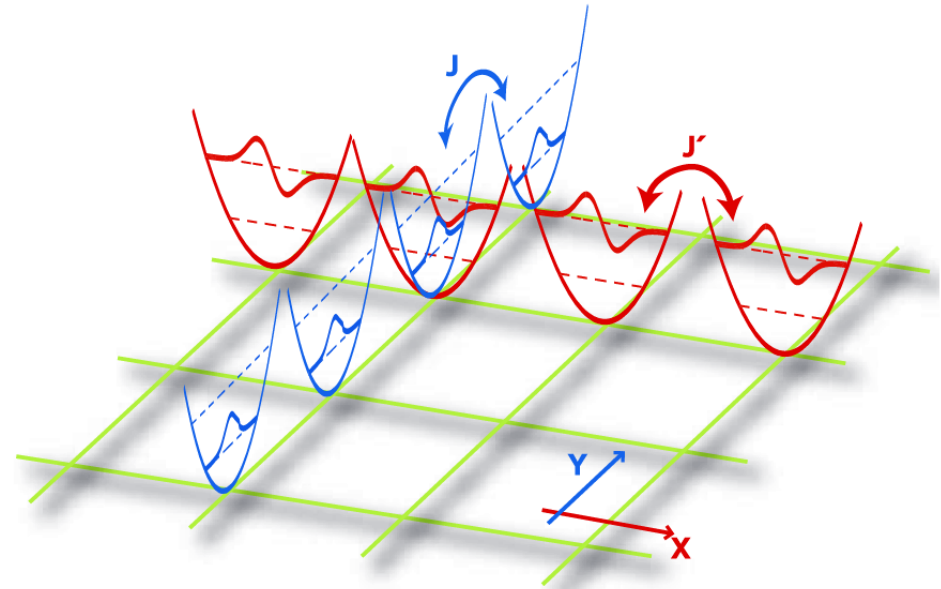
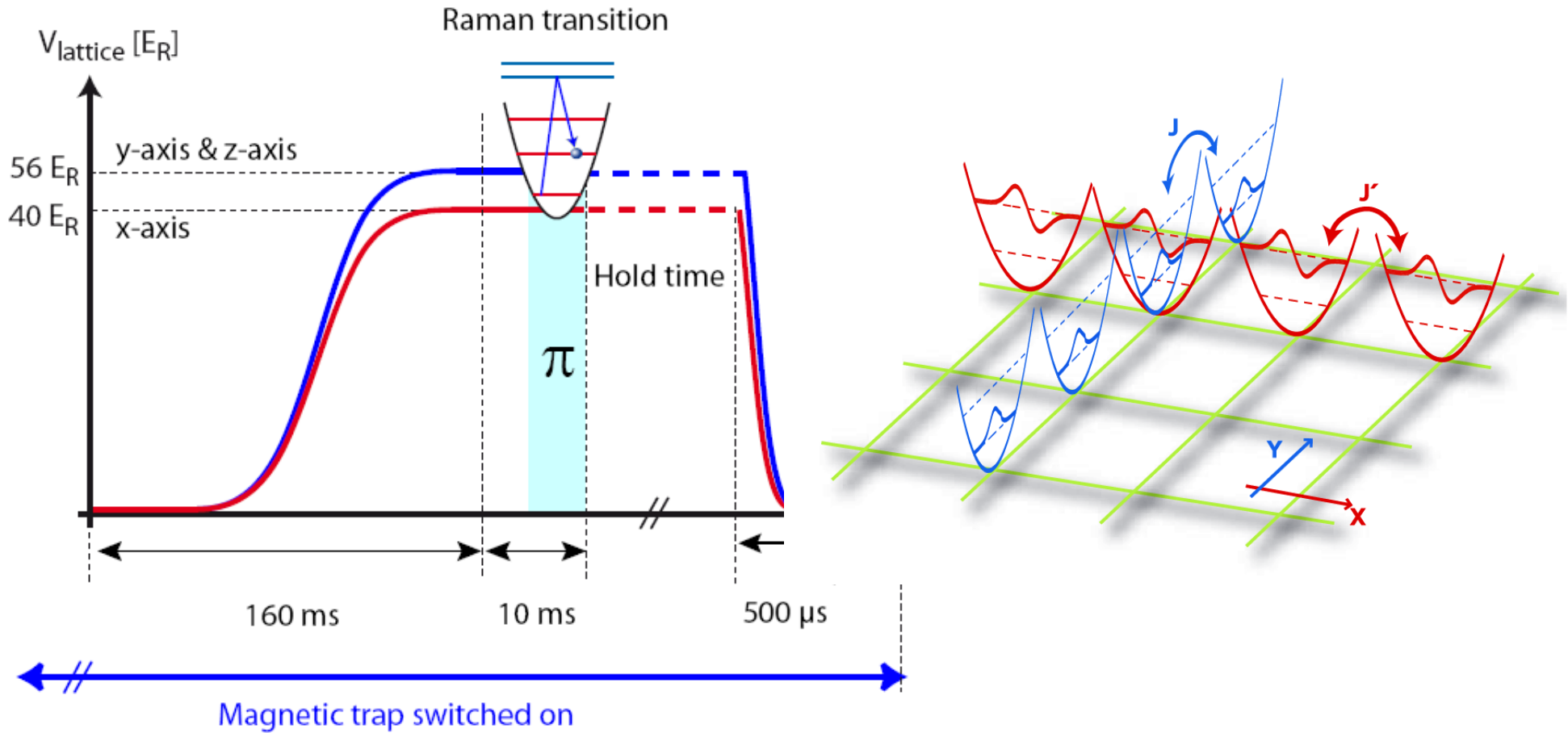
**Note:** hopping and interactions are generically flavor-specific!

[Bloch, Nature Physics (2005)]

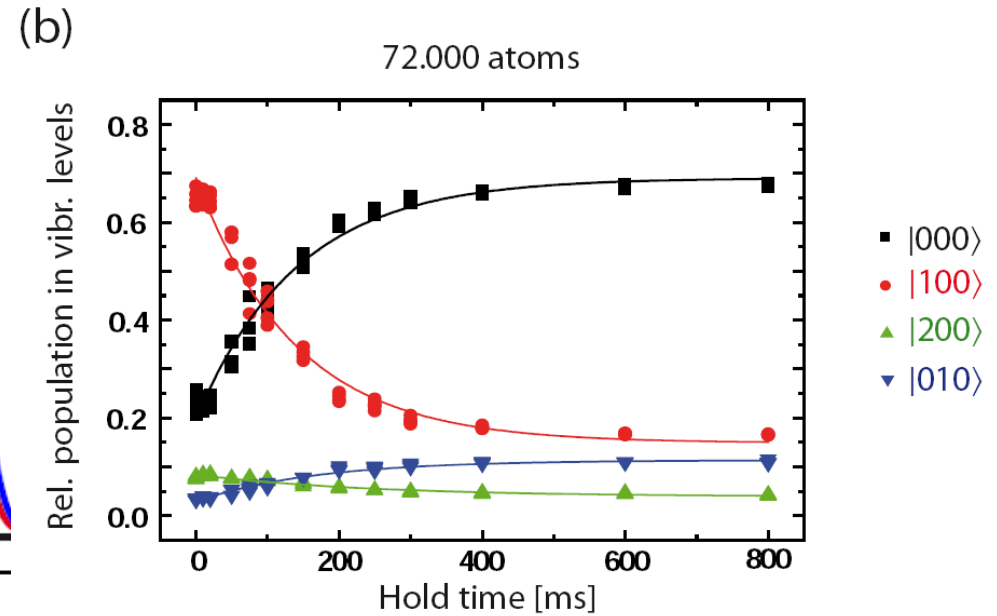
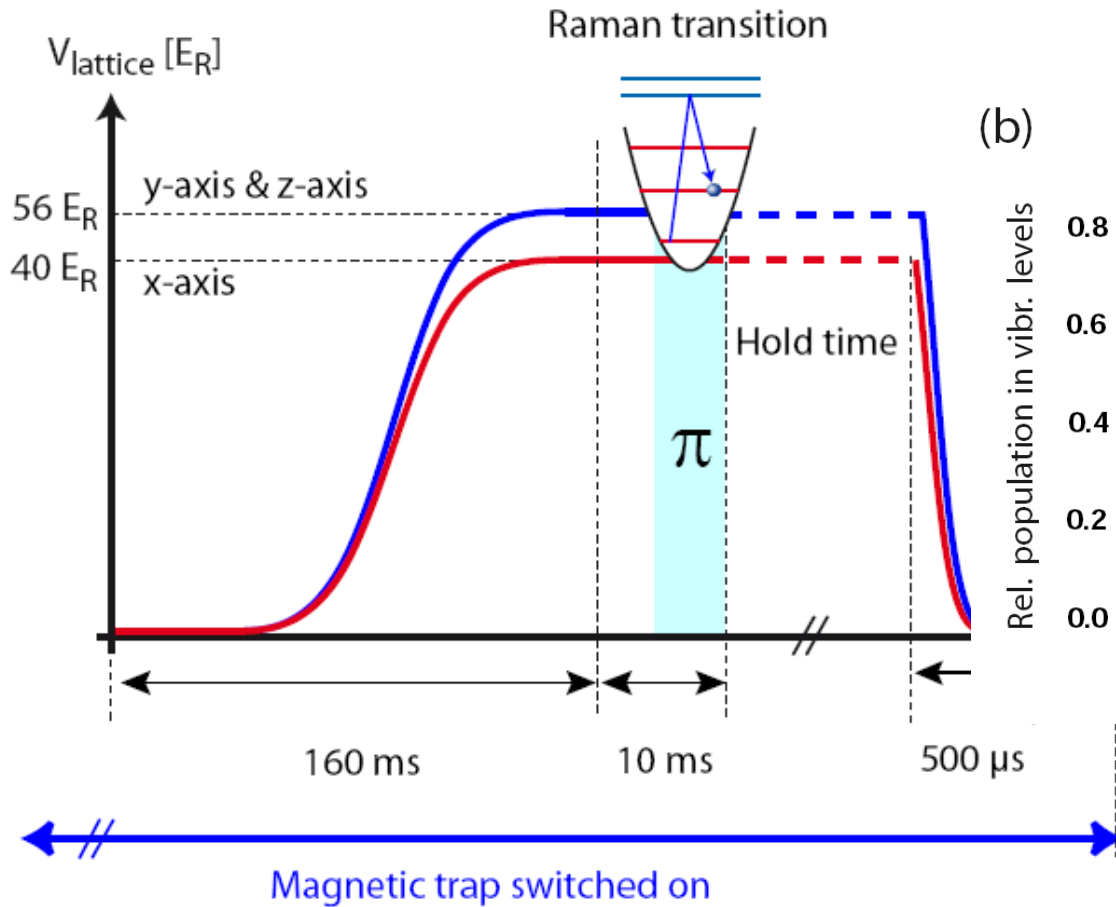
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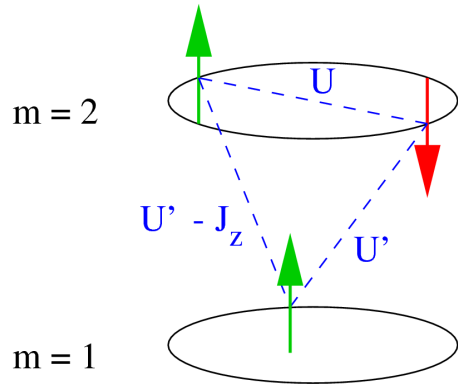
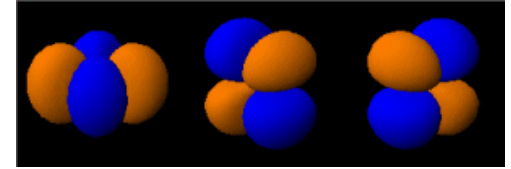
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Many realizations of systems of ultracold atoms on optical lattices with flavor-specific hopping amplitudes (and interactions)!

# OSMTs in 2-band Hubbard model

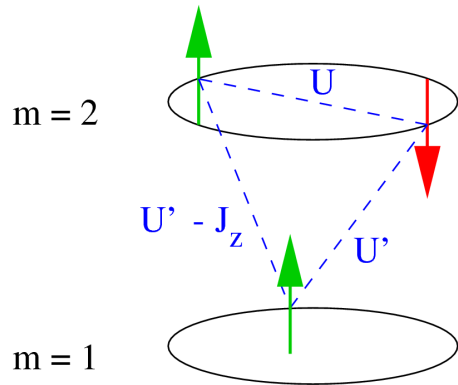
Minimal OSMT model: 2-band Hubbard model with  
with orbital-dependent hopping



$$\begin{aligned}
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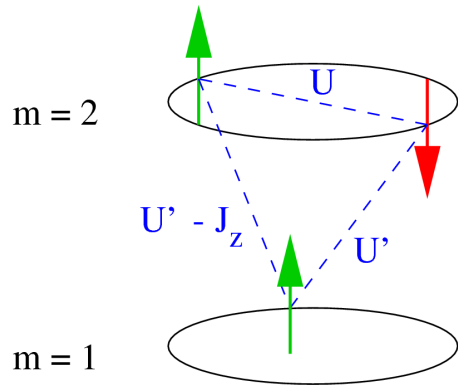
**Note:**  $J_\perp$  causes sign problem in (Hirsch-Fye) QMC

lattice, hopping anisotropic in  $\text{Ca}_{2-x}\text{Sr}_x\text{RuO}_4 \rightsquigarrow$  coupling anisotropy?

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quantum gas: no “spin-flip” terms for different atomic species

Bethe DOS,  $t_2 = 2t_1$ : two distinct MITs for  $J_z = J_\perp = U/4$  [Koga *et al.*, PRL (2004)]

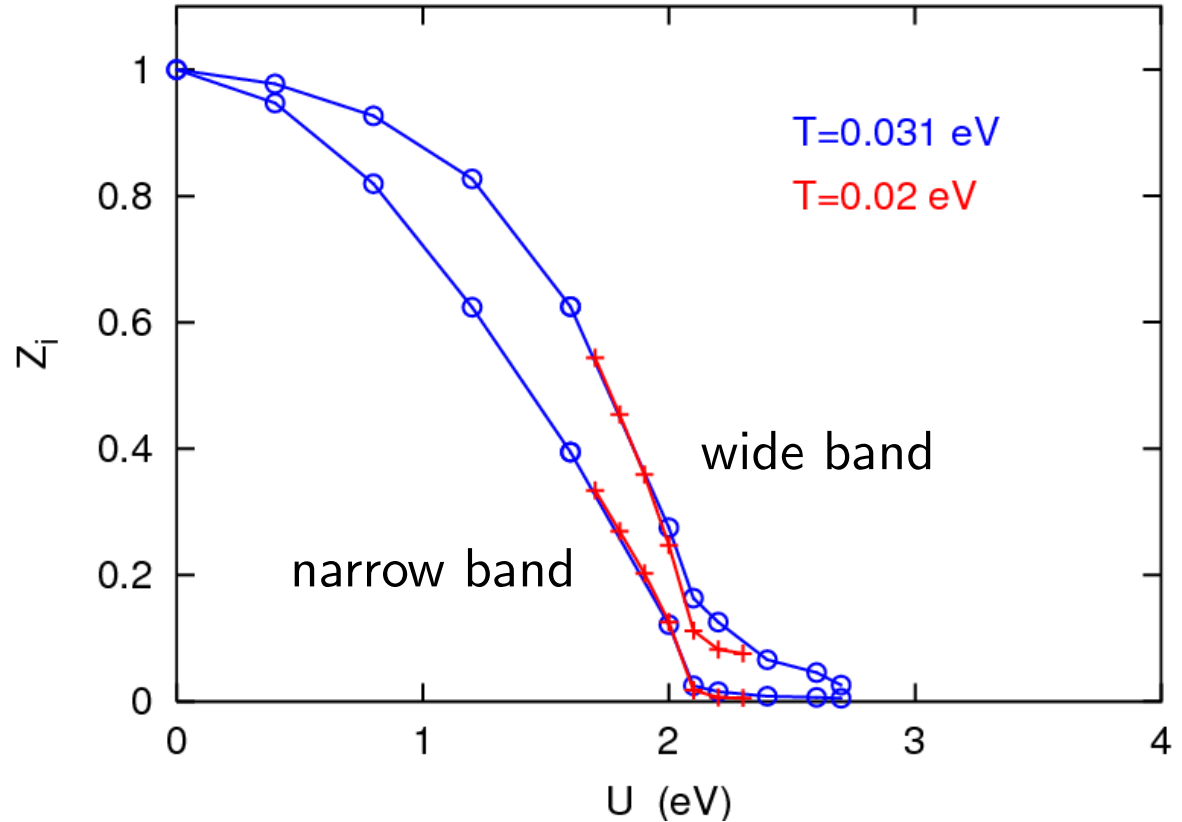
**single MIT** or **two OSMTs** for  $J_z$  model ( $J_\perp = 0$ ) ?

# Early DMFT-QMC results: single Mott transition in $J_z$ model

Choice:  $J_z = U/4$ ,  $U' = U/2$

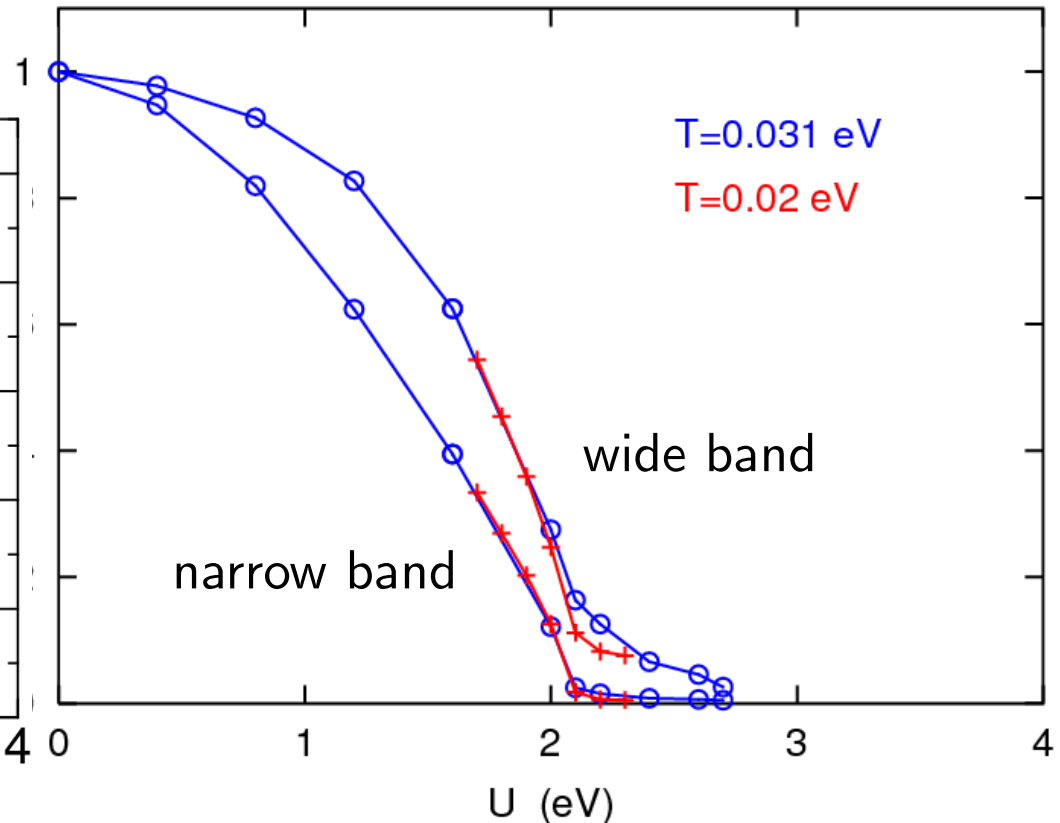
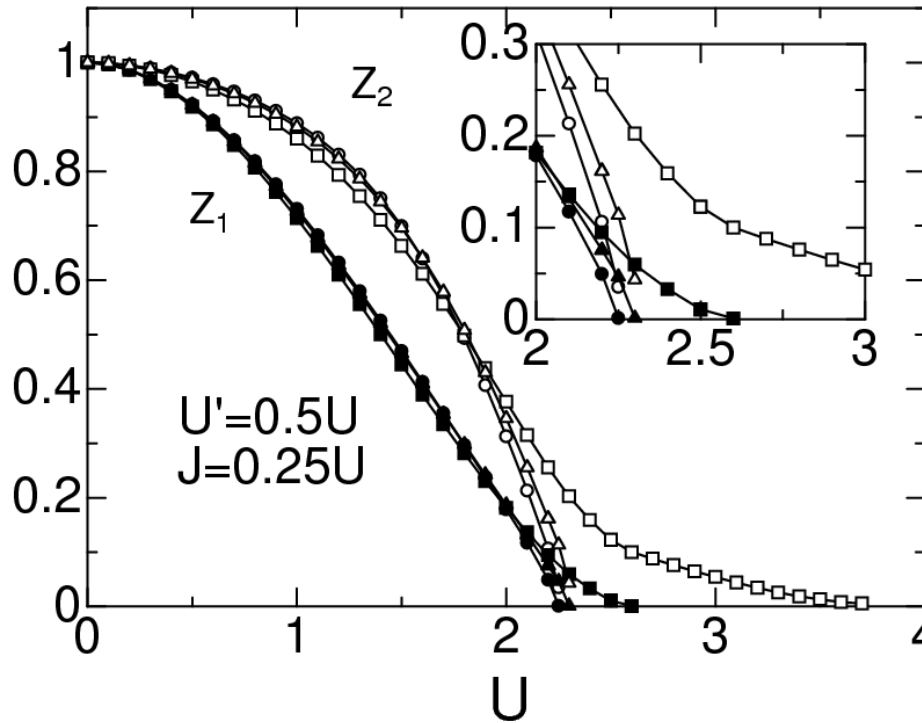
Metallicity  $\sim$  qp weight

$$Z_i = \frac{m}{m^*} \approx [1 - \text{Im}\Sigma(i\pi T)/\pi T]^{-1}$$



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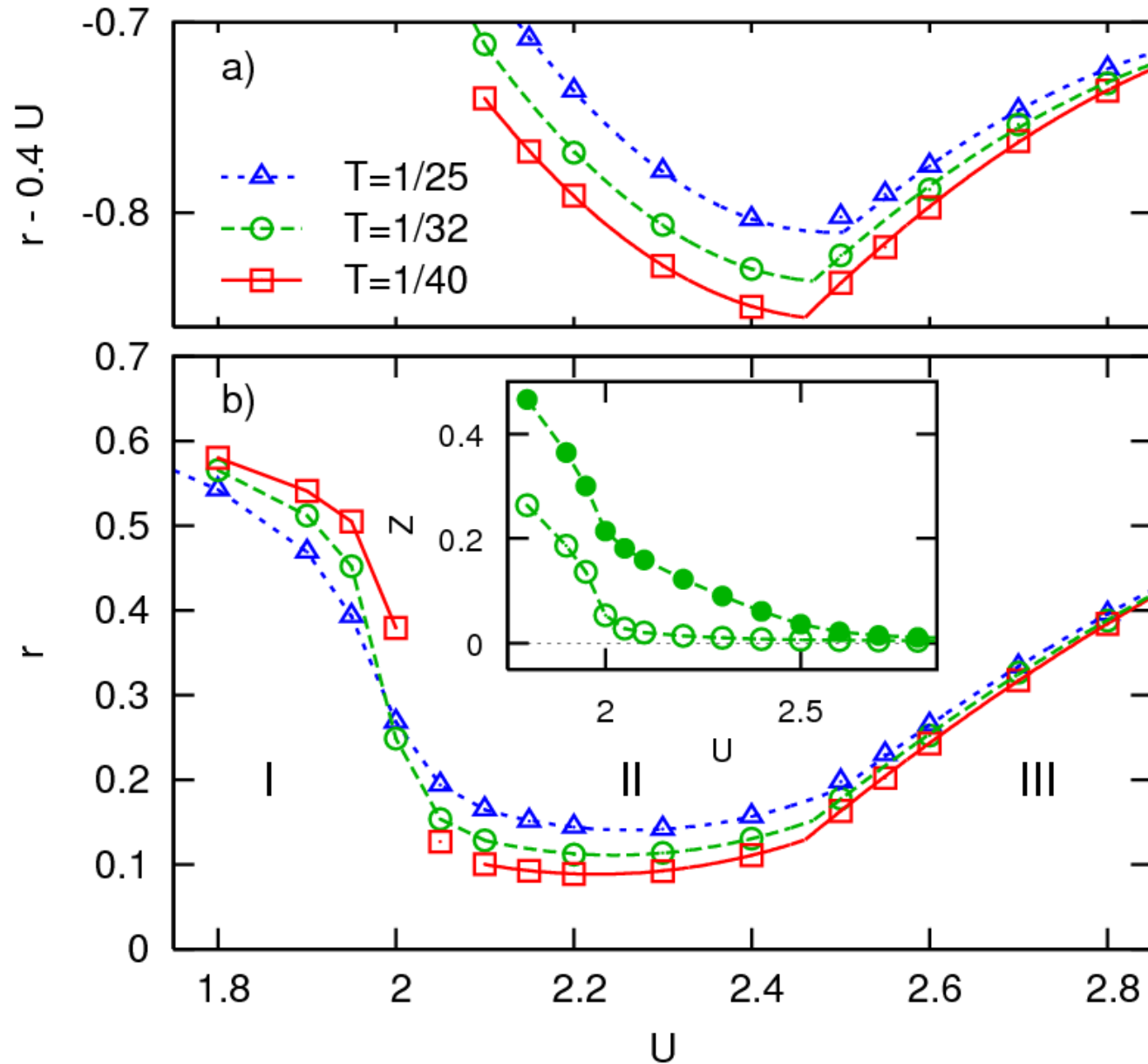


[Koga, Kawakami, Rice, Sigrist, Physica B (2005)] [Liebsch, PRB **70**, 165103 (2004)]

Conclusion as of 03/2005: OSMT scenario in 2-band Hubbard model only for  $SU(2)$ -invariant Hund's exchange ( $J_{\perp} = J_z$ )!

Check using high-precision QMC algorithm . . .

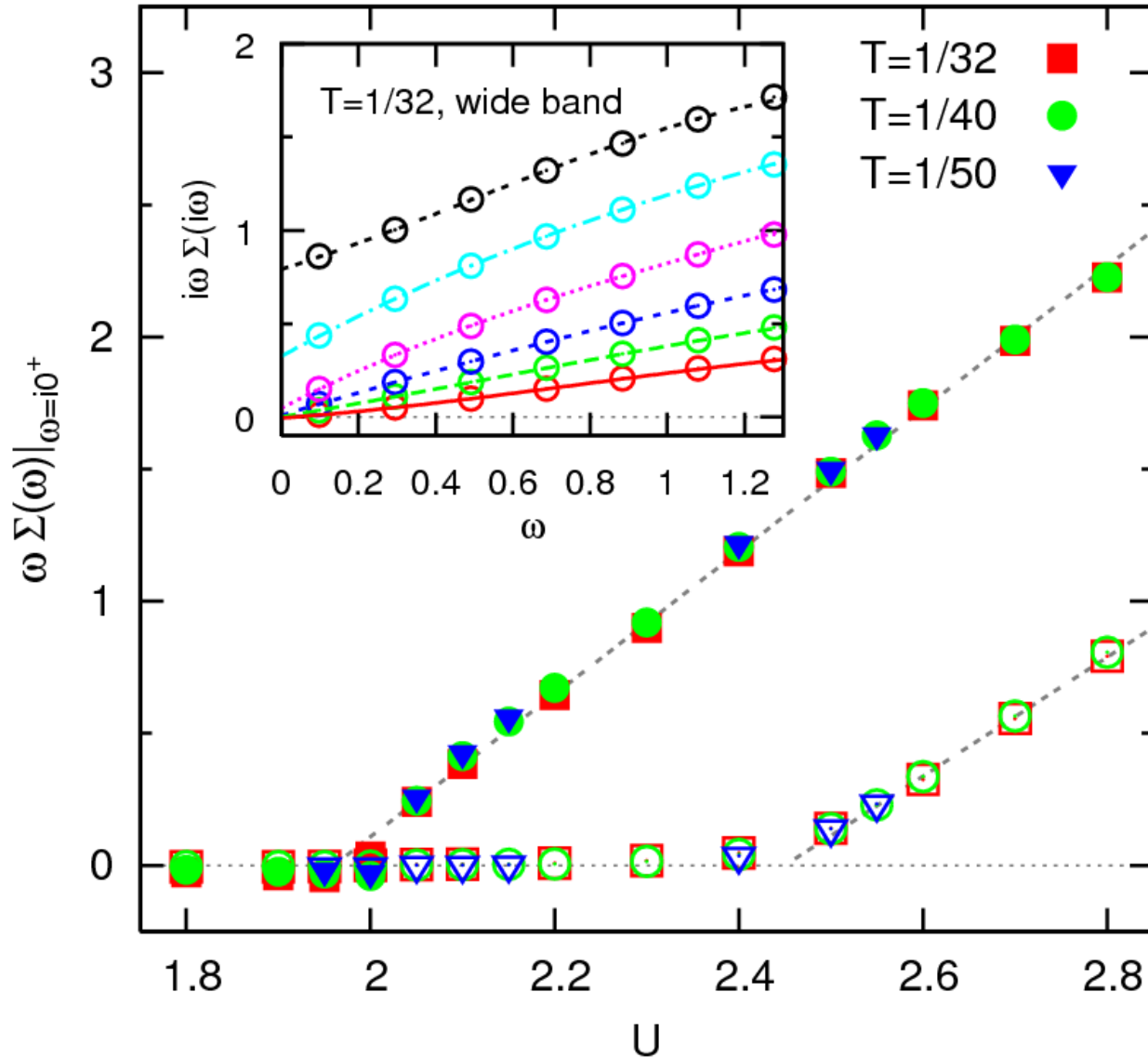
# Ratio of quasiparticle weights $r = Z_{\text{narrow}}/Z_{\text{wide}}$



3 regions of different character

kinks indicate 2<sup>nd</sup> transition at  $U \approx 2.5$

# Low-frequency analysis of self-energy



for regular self-energy:

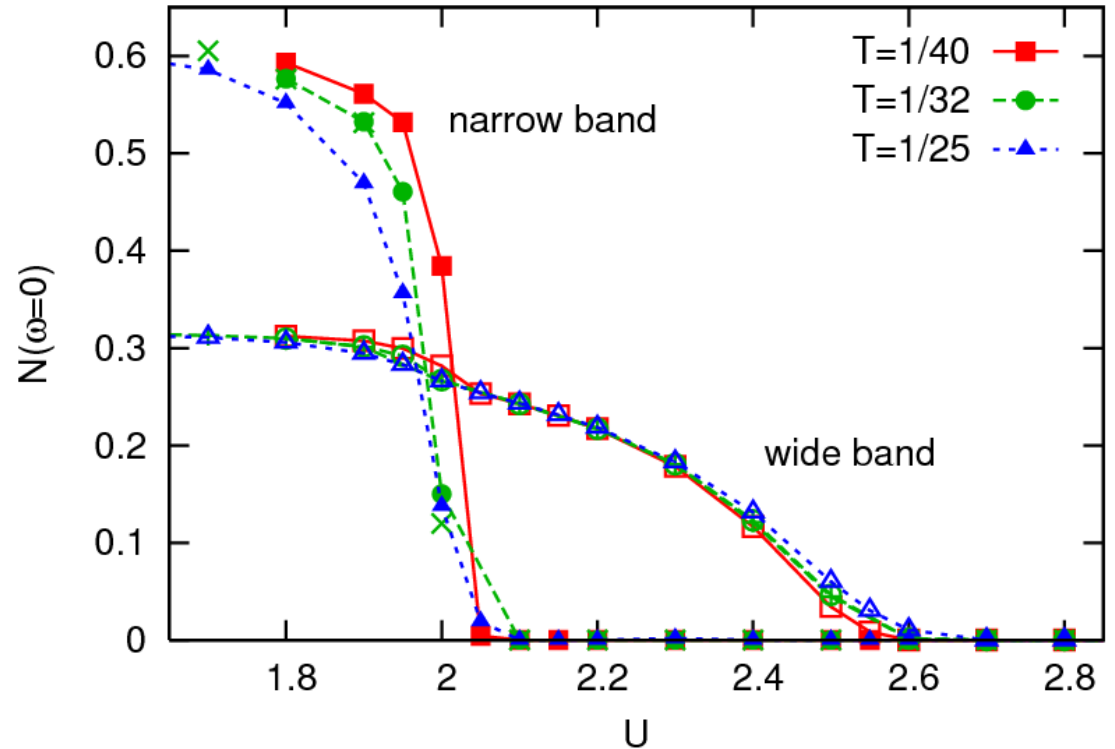
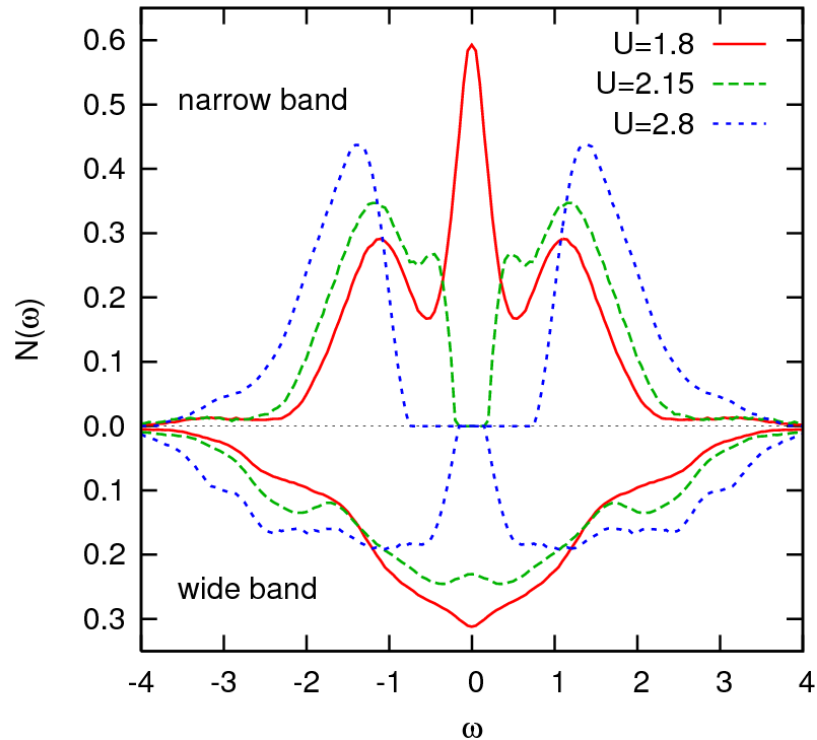
$$\omega \Sigma(\omega) \xrightarrow{\omega \rightarrow 0} 0$$

singularities ( $\sim$  gap) for

$U \gtrsim 2$  in narrow band

$U \gtrsim 2.5$  in wide band

# Spectral function (interacting DOS)



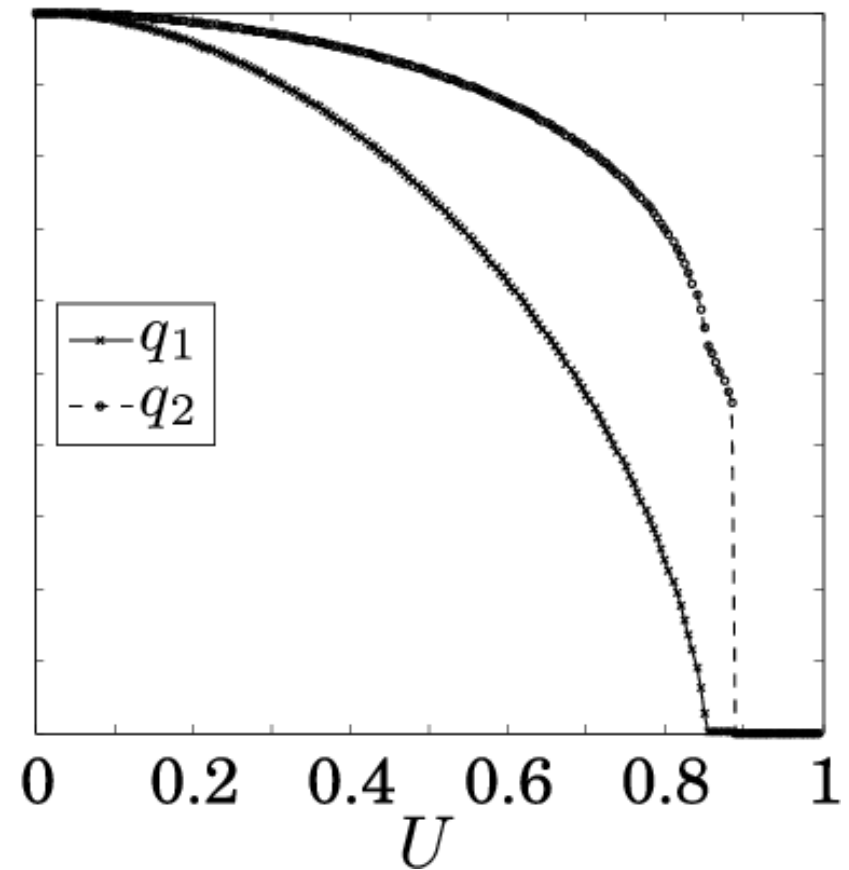
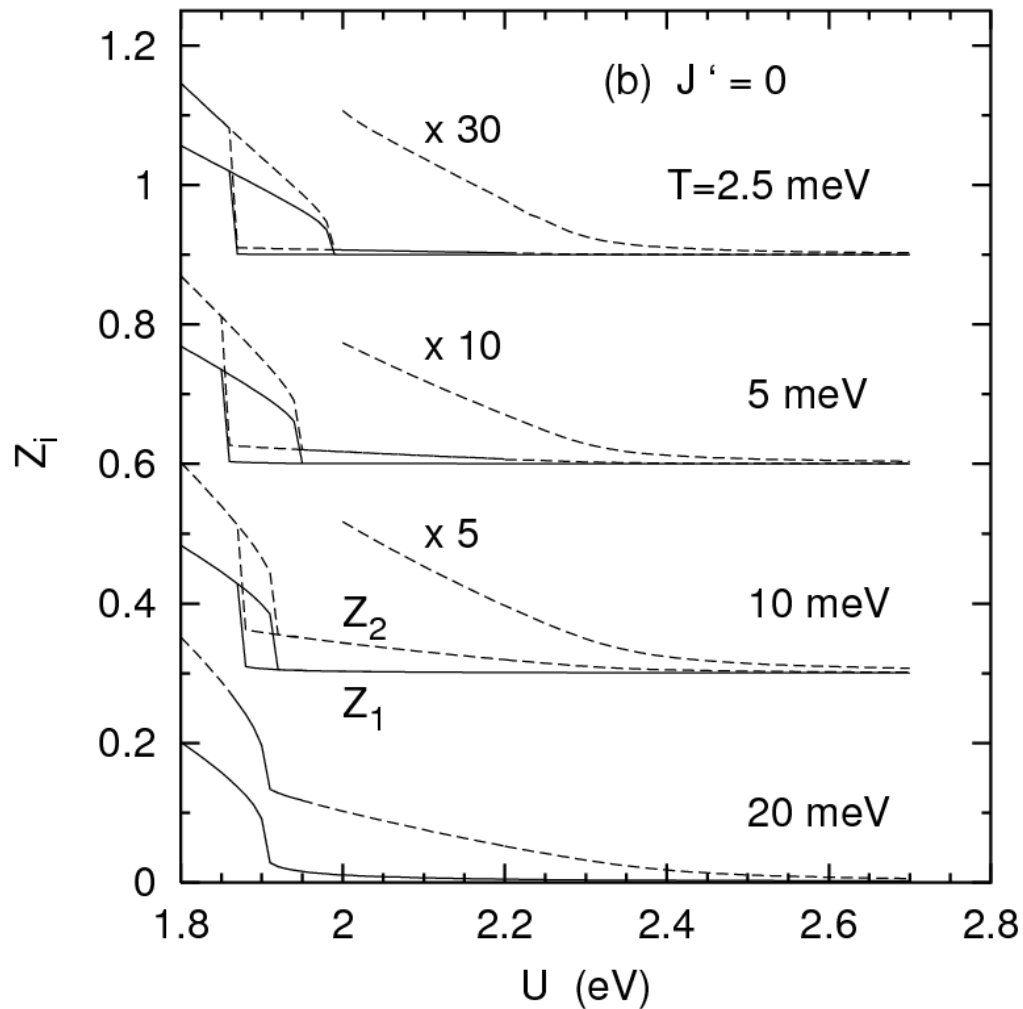
Clear indications for second singularity, wide band remains metallic at  $U \approx 2.0$

↪ **two orbital-selective Mott transitions** [Knecht, NB, van Dongen, PRB (2005)]

same conclusions from slave-spin approx. [de' Medici, Georges, Biermann, PRB (2005)]

and slave-boson MF [Rüegg, Indergand, Pilgram, Sigrist, EPJB (2005)]

# Order of wide-band transition at $T > 0$ and $T = 0$

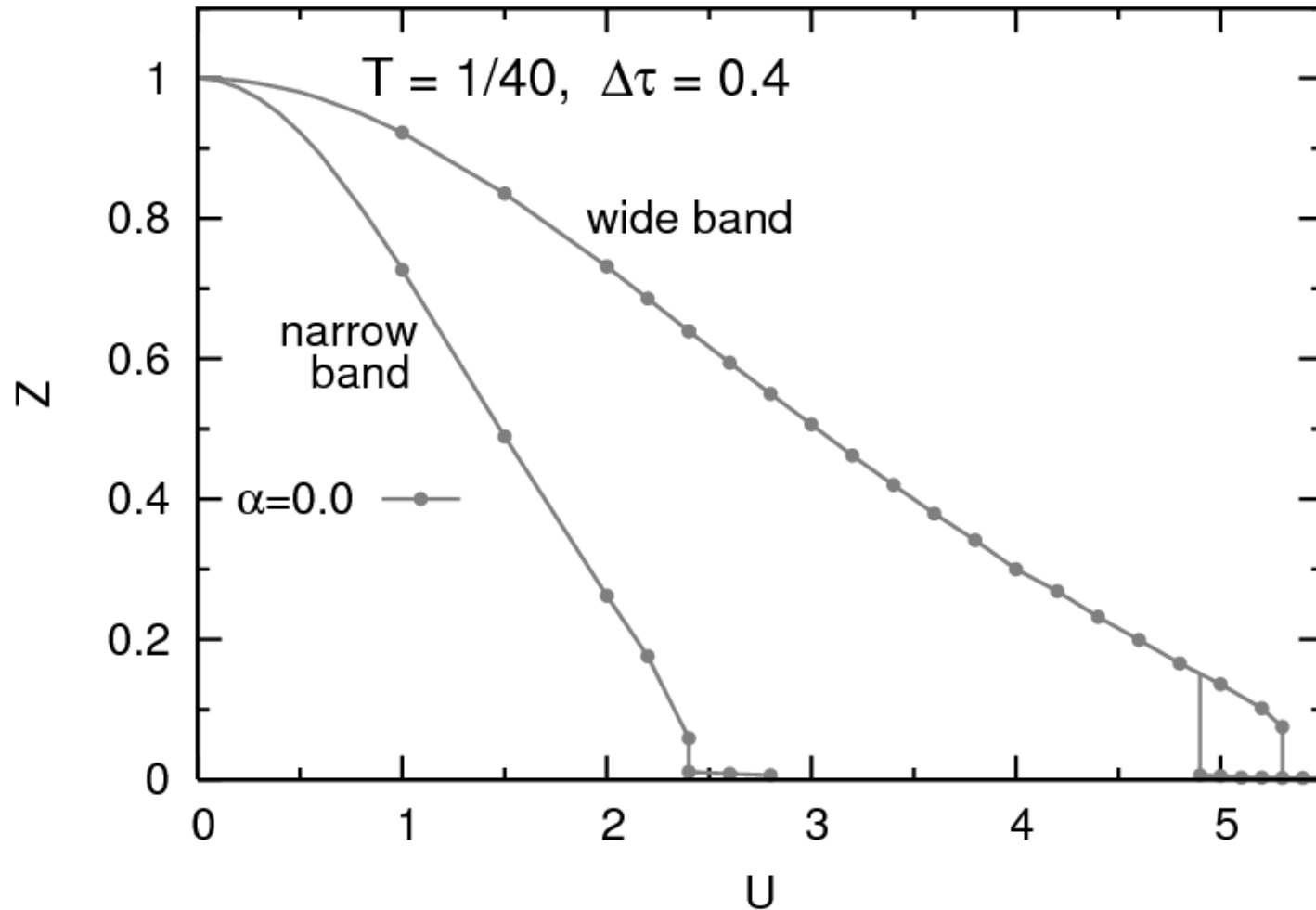


Slave-boson MF  $\rightsquigarrow$  1<sup>st</sup> order wide-band transition (at  $T = 0$ ) [Rüegg, Indergand, Pilgram, Sigrist, EPJB (2005)]

ED  $\rightsquigarrow$  no hysteresis at low  $T$  for wide-band transition [Liebsch, PRL (2005)]

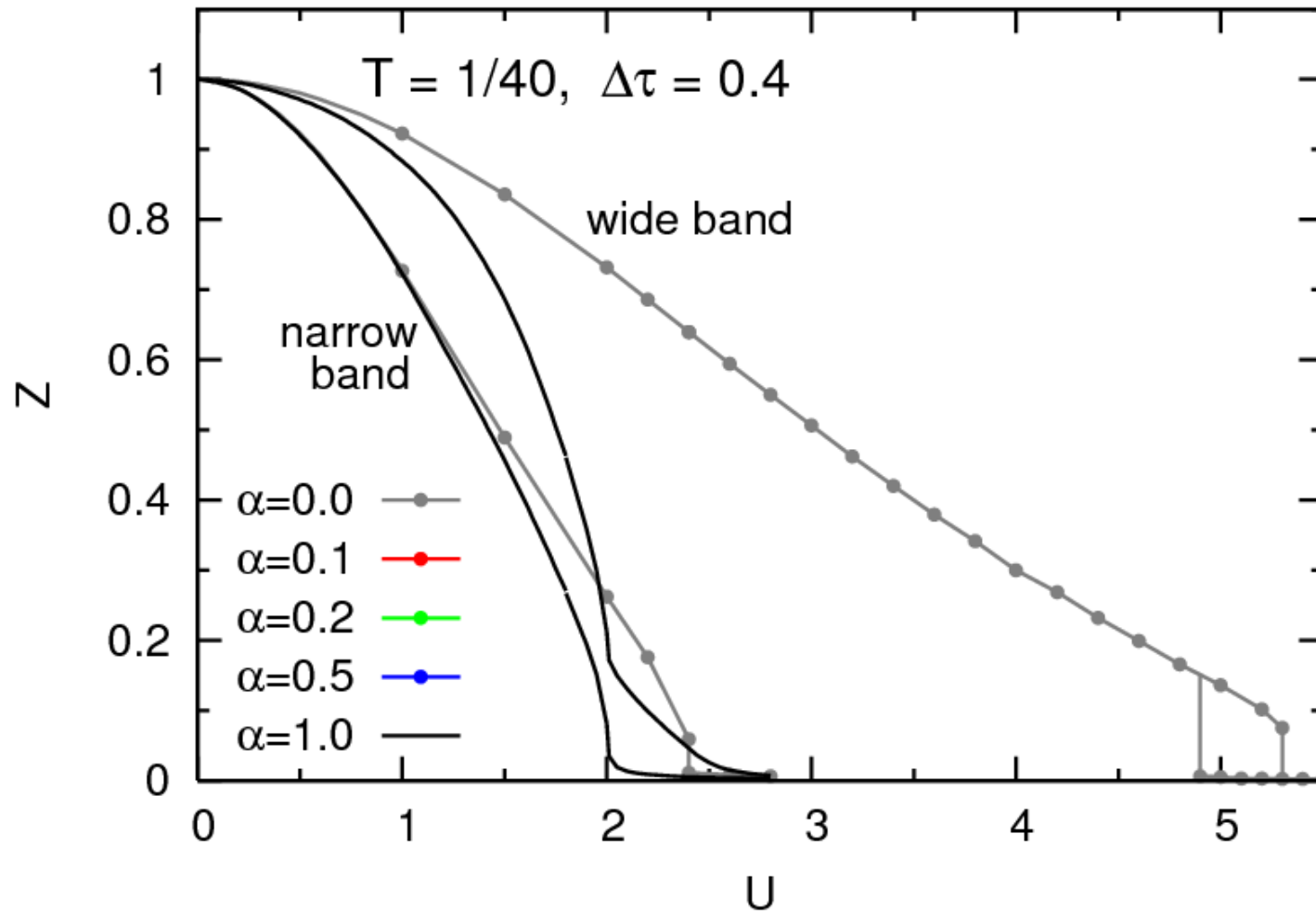
# Systematic study: effect of inter-orbital coupling

$$H = \sum_{m=1}^2 \left[ - \sum_{\langle ij \rangle \sigma} t_m c_{im\sigma}^\dagger c_{jm\sigma} + U \sum_i n_{im\uparrow} n_{im\downarrow} \right] + \alpha \sum_{i\sigma\sigma'} (U/2 - \delta_{\sigma\sigma'} U/4) n_{i1\sigma} n_{i2\sigma'}$$



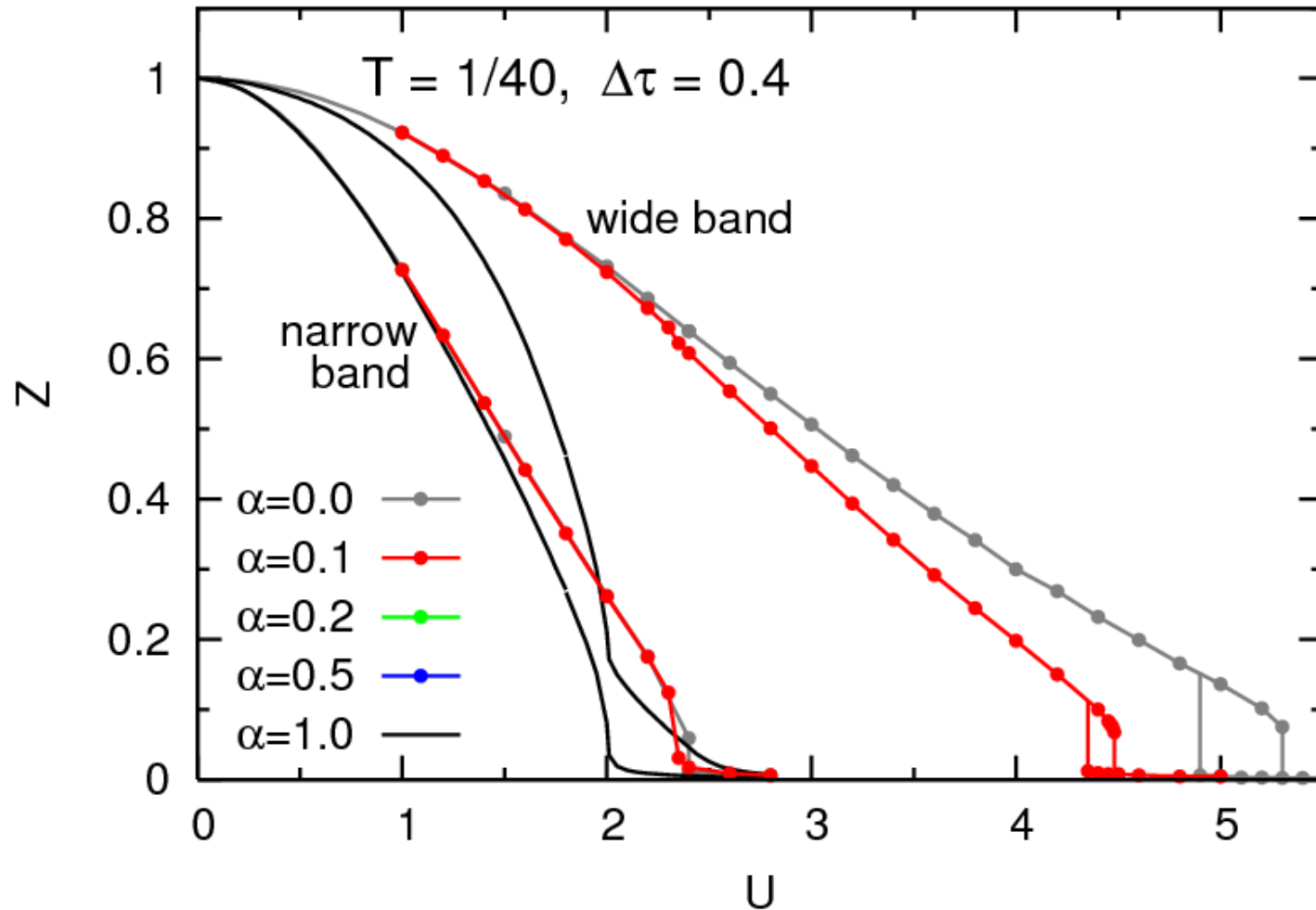
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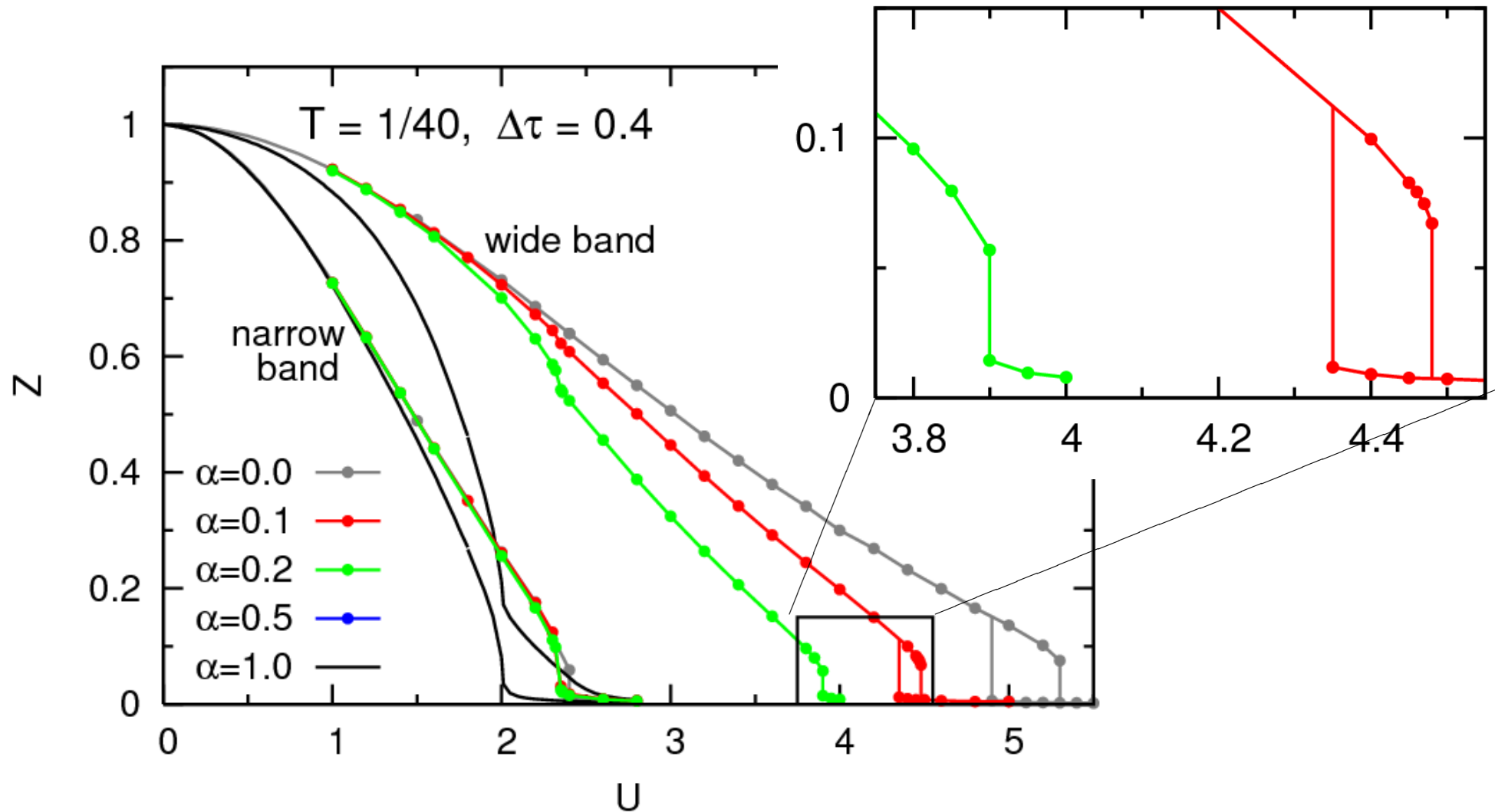
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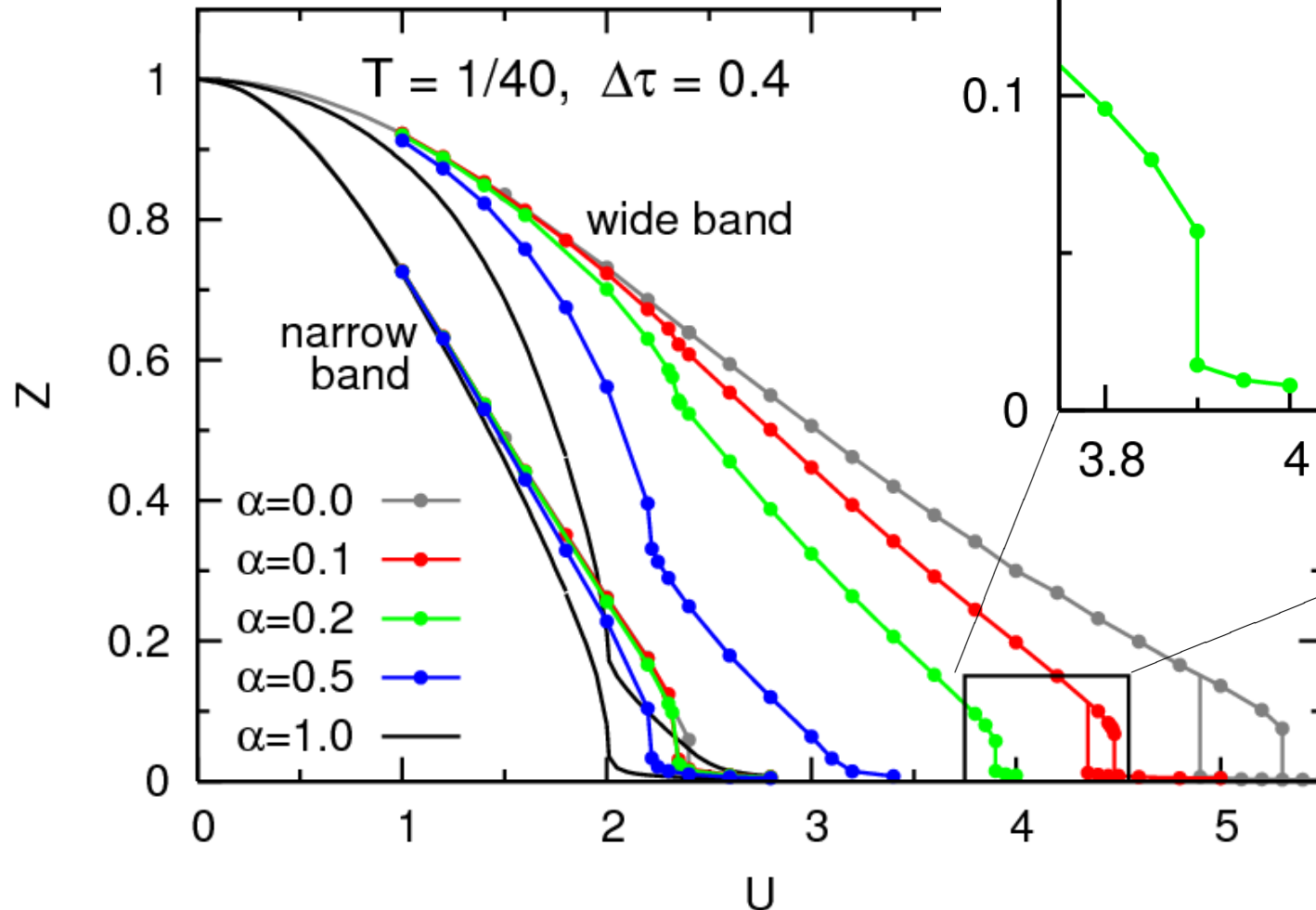
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# Systematic study: effect of inter-orbital coupling

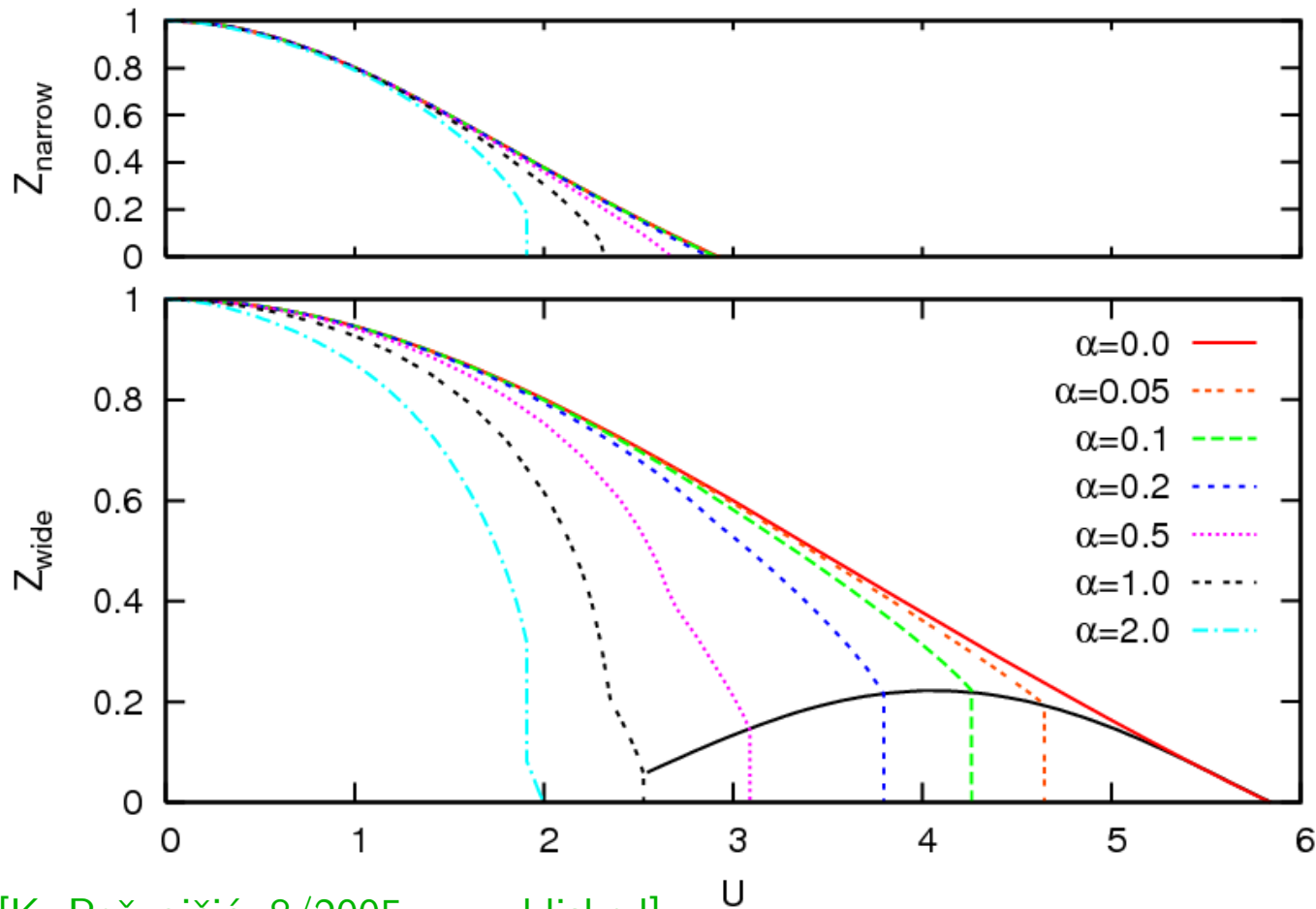
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Wide-band OSMT remains 1<sup>st</sup> order for small  $\alpha$

1<sup>st</sup> order at  $T=0$ ?

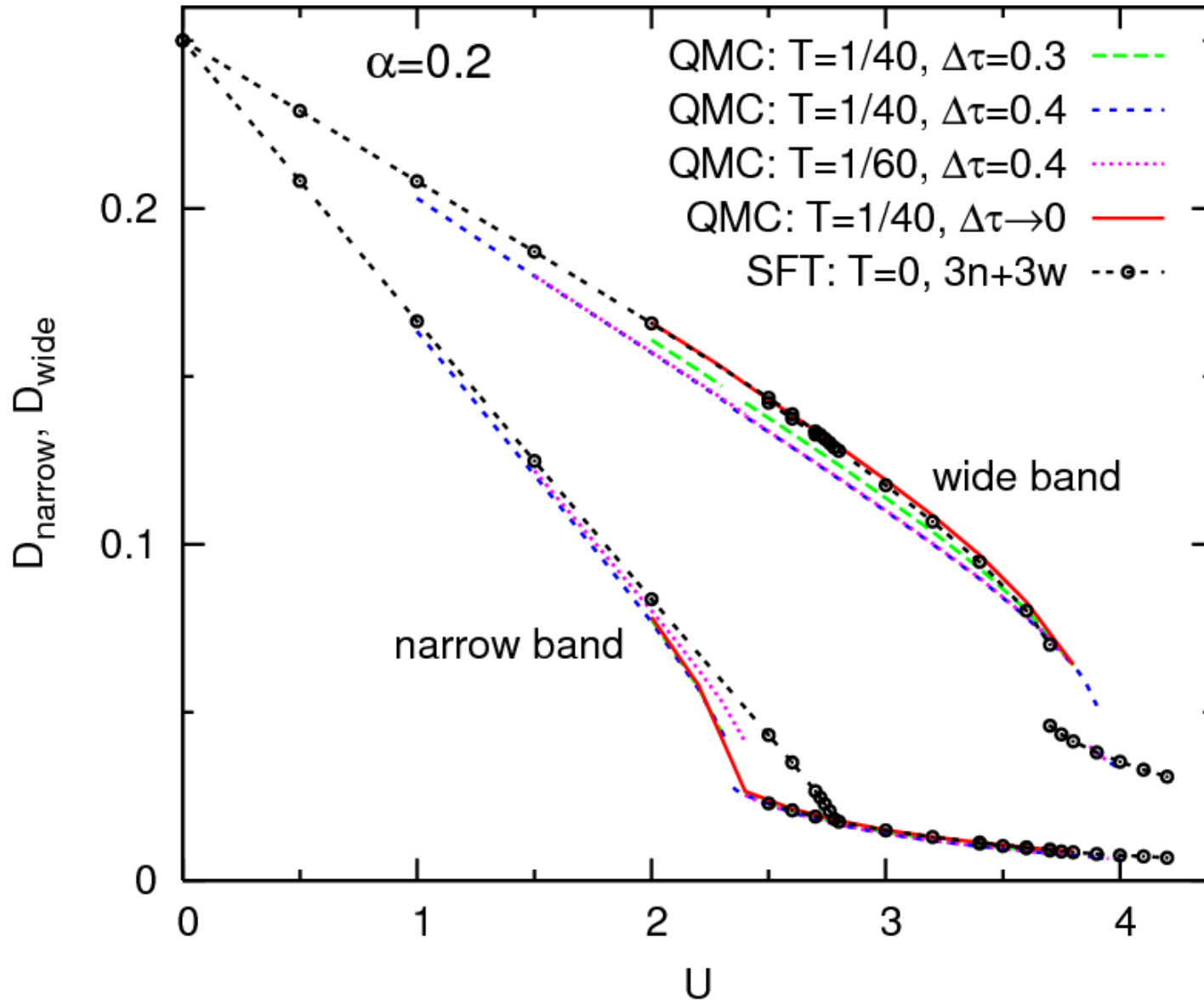
# Self-energy functional theory (SFT+ED) with 1 bath site per orbital



[K. Požgajčić, 8/2005, unpublished]

**Problems:** Low-frequency part of  $\Sigma(\omega)$  inconsistent with QMC  
 $Z$  problematic in OSM phase  $\rightsquigarrow$  search for kinks in  $\Omega$   
massive finite-size effects with increasing number of bath sites!

# Double occupancy (1<sup>st</sup> order derivative of $\Omega$ )



Excellent agreement between SFT and QMC

1<sup>st</sup> order at  $T = 0$  (at least) for  $0 < \alpha < 0.2!$ ?

# Summary

Orbital-selective Mott transitions in 2-band Hubbard model ( $J_z > J_\perp$ )  
transition at  $U \approx 2.0$  is orbital-selective  
clear indications for second singularity at  $U \approx 2.5$

[C. Knecht, NB, P.G.J. van Dongen, Phys. Rev. B **72**, 081103(R) (2005)]

Systematic study: effect of inter-orbital coupling  
wide-band OSMT 1<sup>st</sup> order for small  $\alpha$  for  $T > 0$  and  $T = 0$  (?)

[NB, C. Knecht, K. Požgajčić, P.G.J. van Dongen, JMMM **310**, 922 (2007)]

Not shown: Self-energy and quasiparticle weight  $Z$  in OSM phase  
antiferromagnetic order, Falicov-Kimball model

[P.G.J. van Dongen, C. Knecht, NB, to appear in phys. stat. sol. (b)]

Not shown: many recent results - in particular on SU(2)-invariant Hund coupling  
and non-FL behavior - by Liebsch+Costi, Arita+Held, Koga/Inaba *et al.*, de'Medici  
+Biermann+Georges, Ferrero+Becca+Fabrizio+Capone, . . .

More OSMT physics: FeSi, ( $V_2O_3$ ), fermions on optical lattices . . .