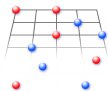


# Double occupancy as a universal probe for antiferromagnetic correlations and entropy in cold fermions on optical lattices

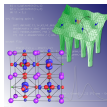
Nils Blümer

Institut für Physik, Johannes Gutenberg-Universität Mainz



TR 49: *Condensed matter systems  
with variable many-body interactions*  
Frankfurt / Kaiserslautern / Mainz

FOR 1346  
LDA+DMFT  
Augsburg *et al.*



Motivation: Ultracold lattice fermions as quantum simulators?

Detection of antiferromagnetic correlations in cold fermions

[Gorelik, Titvinidze, Hofstetter, Snoek, Blümer, PRL (2010)]

Effects of **non-local correlations**? DMFT versus direct QMC + BA

[Gorelik, Paiva, Scalettar, Klümper, Blümer, arXiv:1105.3356]

Realistic modeling: **anisotropic trap** (“slab” method  $\sim$  GGA)

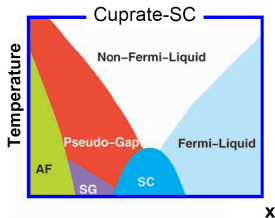
[Gorelik, Blümer, to appear in JLTP]

Characteristic temperature of **pseudogap** in  $2d$  Hubbard model?

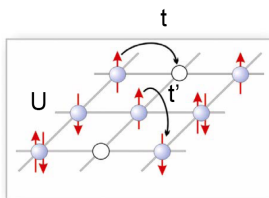
Summary and outlook

# Motivation: Ultracold lattice fermions as quantum simulators?

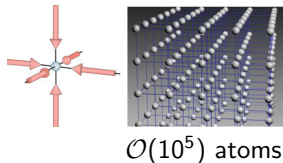
## Correlated materials



## Fermionic Hubbard model



## Ultracold fermions on optical lattices



$\mathcal{O}(10^5)$  atoms

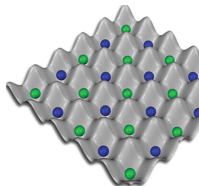
## Recent breakthrough: paramagnetic Mott transition in 2-flavor mixtures

[Schneider et al., *Science* **322**, 1520 (2008), Jördens et al., *Nature* **455**, 204 (2008)]

## Remaining challenge: antiferromagnetism (staggered order)

### Problems:

- (i) difficult to reach sufficiently low temperatures/entropies
- (ii) detection of AF order is not straightforward
- (iii) inhomogeneity, time scale for global (spin) equilibrium

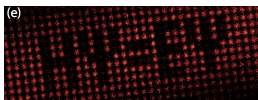


## Questions for first part of this talk

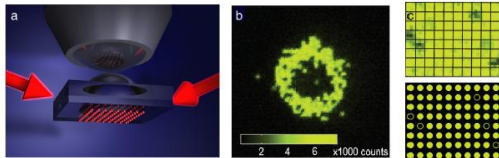
- How to detect AF order/correlations?
- Which entropy range is needed?
- **General impact of dimensionality?**

Mermin-Wagner: LRO  $\leftrightarrow d = 3$

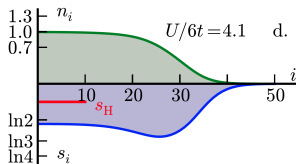
Experimental advantage of 2 dimensions:  
single-site resolution (for bosons)



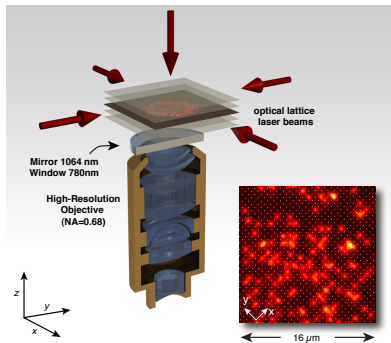
[Würtz et al., PRL **103**, 080404 (2009)]



[Bakr et al., Science **329**, 547 (2010)]



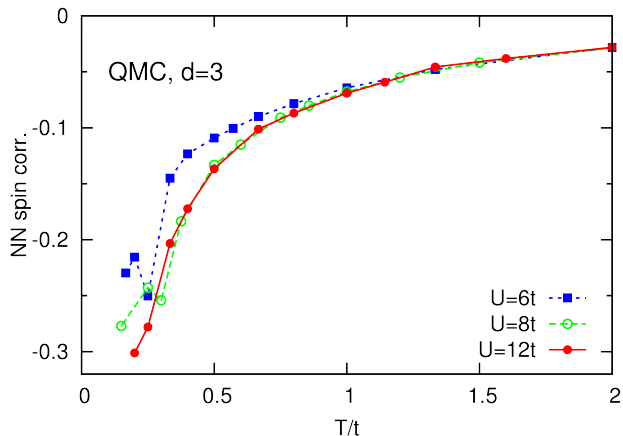
[Jördens et al., PRL **104**, 180401 (2010)]



[Sherson et al., Nature **467**, 68 (2010)]

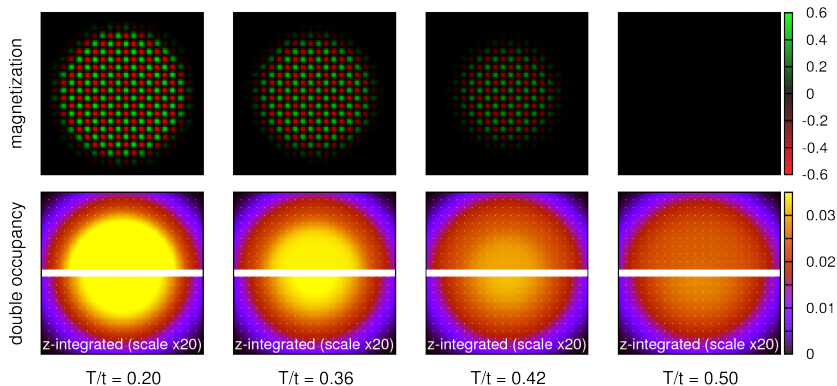
## Current experimental focus: nearest-neighbor spin correlation function

Modulation spectroscopy (Esslinger group), super-lattice (Bloch group)



Note: strong (universal) high-temperature tails, monotonous  
no distinct features at Néel temperature ( $\approx 0.3t$ )

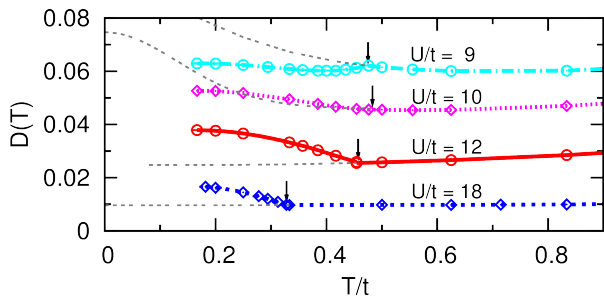
# Background: RDMFT-QMC (cubic lattice, $V = 0.05t$ , $U = W = 12t$ )



Proposal: enhanced double occupancy (i.e. interaction energy) as a signature of antiferromagnetic order at strong coupling

[Gorelik, Titvinidze, Hofstetter, Snoek, Blümer, PRL (2010)]

# DMFT-QMC estimates of double occupancy $D$ at half filling



At  $U \gtrsim 10t$ :

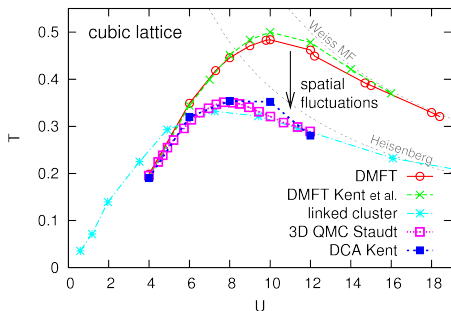
$D$  enhanced below Néel temperature (arrows)

Thin lines: metastable nonmagnetic phase

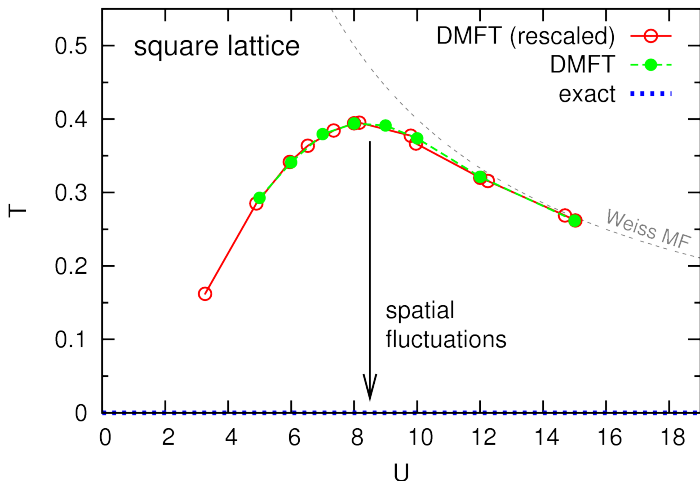
Problem: DMFT overestimates ordering temperatures

$\rightsquigarrow$  kinks cannot remain at

$$T = T_N^{\text{DMFT}} > T_N!$$



Situation “worse” in 2 dimensions:  $T_N = 0 \ll T_N^{\text{DMFT}}!$



Mermin-Wagner theorem excludes AF long-range order at finite temperatures

# Effects of non-local correlations? Comparisons with direct QMC + BA

[Gorelik, Rost, Paiva, Scalettar, Klümper, NB, arXiv:1105.3356]



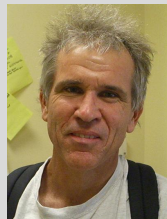
Elena Gorelik  
Univ. Mainz



Andreas Klümper  
Univ. Wuppertal

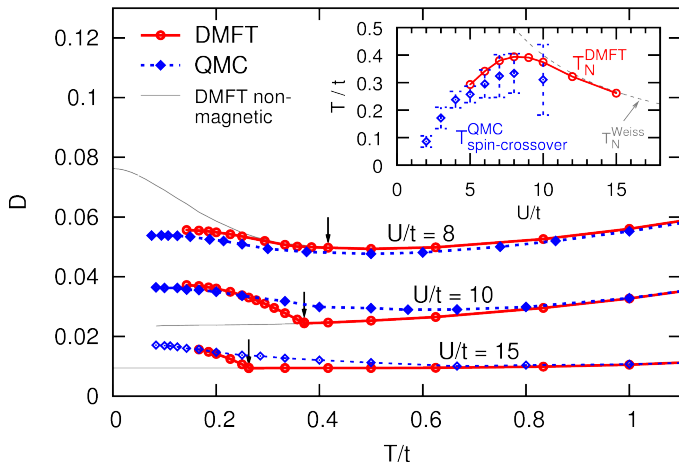


Thereza Paiva  
Rio de Janeiro



Richard Scalettar  
UC Davis

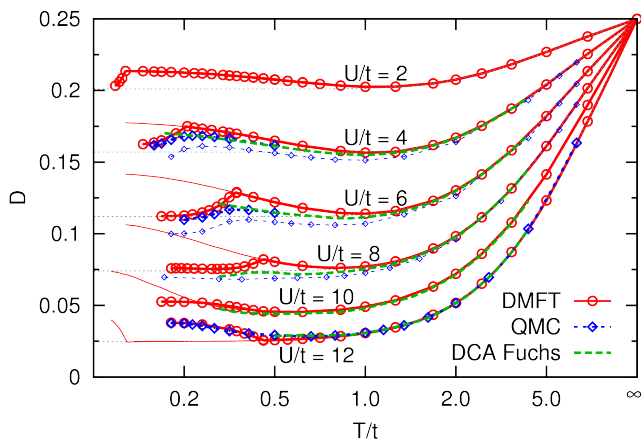
# Comparison DMFT – direct QMC for the 2d square lattice ( $n = 1$ )



AF DMFT confirmed at high  $T$  and at low  $T$ , rounding off at  $T \approx T_N^{\text{DMFT}}$

Nonmagnetic DMFT completely off at low  $T$ !!!

## Comparison DMFT – direct QMC for the 3d cubic lattice ( $n = 1$ )

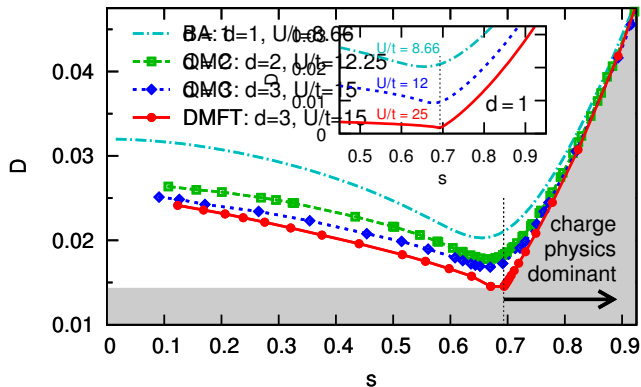


Excellent general agreement DMFT  $\leftrightarrow$  QMC, even at small  $U$

DCA study [Fuchs et al., PRL (2011)] misses AF signatures

Typical QMC discretization errors (thin lines) larger than DMFT deviations!

# Double occupancy as a universal measure of AF correlations + entropy



AF enhancement of  $D$  is larger

in lower dimensions:

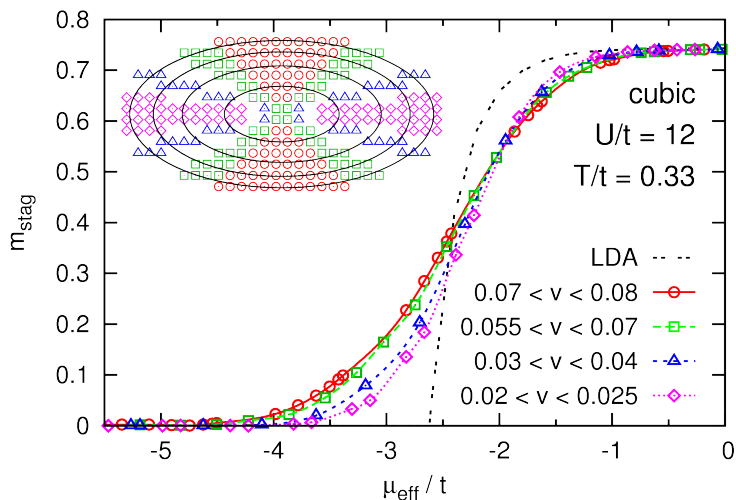
$$D_0 = (1 - \langle \sigma_i \cdot \sigma_j \rangle) Z \frac{t^2}{2U^2} + \mathcal{O}(t^4/U^4)$$

$$\langle \sigma_i \cdot \sigma_j \rangle_0 = \begin{cases} -1.00 & \text{DMFT} \\ -1.20 & (d = 3) \\ -1.34 & (d = 2) \\ -1.77 & (d = 1) \end{cases}$$

# Realistic modeling: anisotropic trap

[Gorelik, Blümer, to appear in JLTP]

# RDMFT for anisotropic trap (aspect ratio 4:1)



Quantification of trap anisotropy: 
$$v = \frac{|\nabla V|^2}{4V} = \frac{V_{\parallel}^2(z/a)^2 + V_{\perp}^2(\rho/a)^2}{V_{\parallel}(z/a)^2 + V_{\perp}(\rho/a)^2},$$

# Characteristic temperature of pseudogap in the two-dimensional Hubbard model?

[Preliminary results]



Daniel Rost  
Univ. Mainz



Fakhri Assaad  
Univ. Würzburg

# Pseudogap in square lattice Hubbard model from DQMC

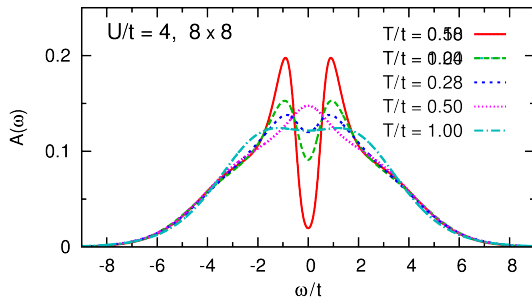
Spectral function (test case for quasi-CT determinantal DQMC and MEM)

At weak coupling ( $U = 4t$ ):

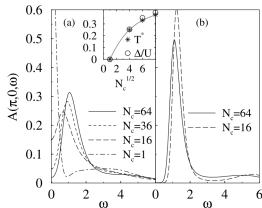
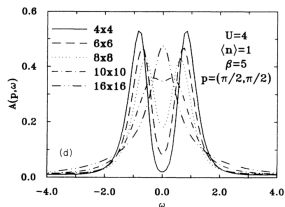
FL peak at intermediate  $T$

Pseudogap for  $T \lesssim T_N^{\text{DMFT}}$

– coincidence?



# Existence and nature of pseudogap: long controversy!

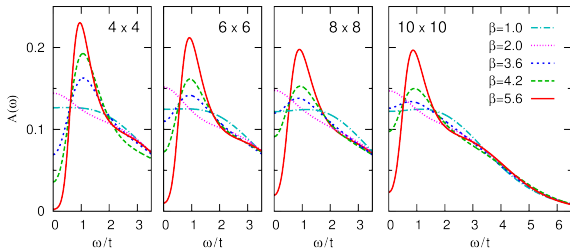


Gap is finite-size effect

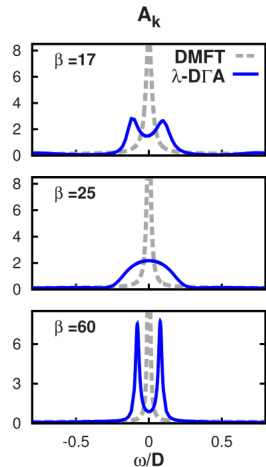
[White, PRB (1992)]

DCA/QMC  $\rightsquigarrow$  gap

[Huscroft et al., PRL (2001)]



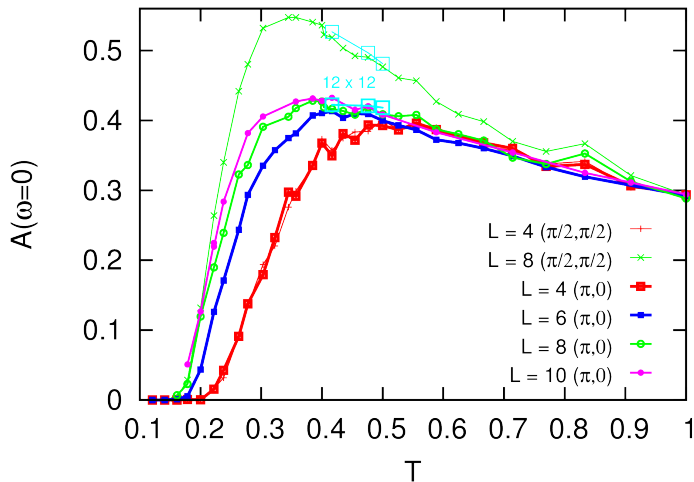
Sweeps +  $\Delta\tau$  important; moderate finite-size effects



DΓA: reentrant behavior

[Katanin et al., PRB (2009)]

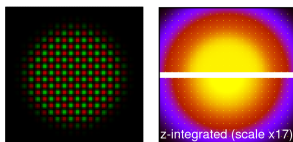
## Clue to finite-size extrapolation: shifting cross-over temperature



Crossover-temperature seems to converge to  $T \gtrsim T_N^{\text{DMFT}} \approx 0.25t!$

Same picture as for double occupancy!

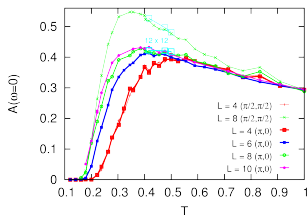
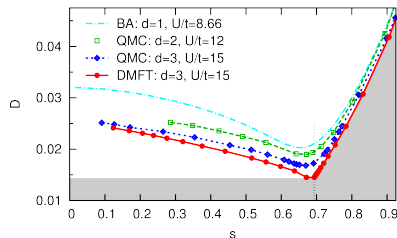
# Summary



**RDMFT**: accurate approach for inhomogeneous correlated Fermi systems (cold atoms or materials)

**Double occupancy**: universal probe of AF correlations and entropy

Relevant **entropy scale** for ultracold experiments (local probes):  $s \approx \log(2)$



**Characteristic pseudogap temperature** in half-filled 2-dimensional Hubbard model

## Outlook

New capabilities: entropy, dynamical information, anisotropy, DQMC

Methodological exchange with Hofstetter group (impurity solvers)

Experimental **signatures of antiferromagnetic correlations**

Impact of **dimensionality** (DMFT versus DQMC)

**Frustration**: cold atoms and **materials**

**Inhomogeneities**, e.g. impurity atom in harmonic trap

**Inequivalent flavors** ( $\sim$  orbital-selective Mott transitions), **multi-flavor**