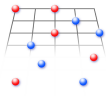


# Double occupancy as a universal probe for antiferromagnetic correlations and entropy in cold fermions on optical lattices

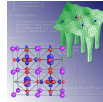
Nils Blümer

Institut für Physik, Johannes Gutenberg-Universität Mainz



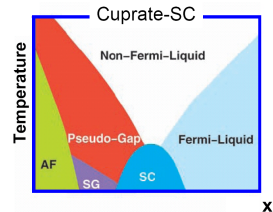
TR 49: *Condensed matter systems with variable many-body interactions*  
Frankfurt / Kaiserslautern / Mainz

FOR 1346  
LDA+DMFT  
Augsburg *et al.*

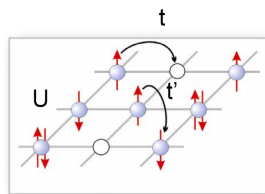


## Motivation: Ultracold lattice fermions as quantum simulators?

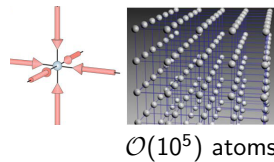
Correlated materials



Fermionic Hubbard model



Ultracold fermions on optical lattices



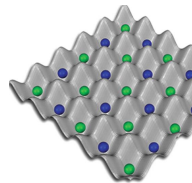
Recent breakthrough: [paramagnetic Mott transition in 2-flavor mixtures](#)

[Schneider *et al.*, *Science* **322**, 1520 (2008), Jördens *et al.*, *Nature* **455**, 204 (2008)]

Remaining challenge: [antiferromagnetism](#) (staggered order)

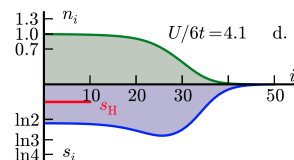
Problems:

- (i) difficult to reach sufficiently [low temperatures/entropies](#)
- (ii) [detection](#) of AF order is not straightforward
- (iii) inhomogeneity, [time scale](#) for global (spin) equilibrium



## Questions for this talk

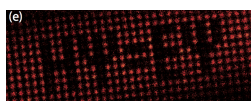
- o How to detect AF order/correlations?
- o Which entropy range is needed/interesting?
- o [General impact of dimensionality?](#)



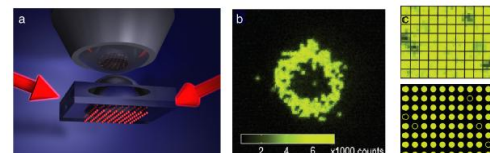
[Jördens *et al.*, *PRL* **104**, 180401 (2010)]

Mermin-Wagner: LRO  $\leftrightarrow d = 3$

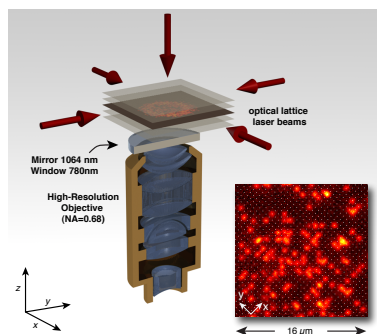
Experimental advantage of 2 dimensions:  
single-site resolution (for bosons)



[Würtz *et al.*, *PRL* **103**, 080404 (2009)]



[Bakr *et al.*, *Science* **329**, 547 (2010)]



[Sherson *et al.*, *Nature* **467**, 68 (2010)]

## Outline

Motivation: Ultracold lattice fermions as quantum simulators?

How to detect AF order/correlations? Is NN spin correlation useful?

Néel transition of lattice fermions in a harmonic trap

[Gorelik, Titvinidze, Hofstetter, Snoek, Blümer, PRL (2010)]

Effects of non-local correlations? DMFT versus direct QMC + BA

[Gorelik, Paiva, Scalettar, Klümper, Blümer, arXiv:1105.3356]

Other observables: energetics, longer-range correlations; weak coupling

Summary and outlook

## How to detect AF order/correlations?

Obvious choice: order parameter

$$m_{stag}^z = \frac{1}{N} \left[ \sum_{i \in A} \langle n_{i\uparrow} - n_{i\downarrow} \rangle - \sum_{i \in B} \langle n_{i\uparrow} - n_{i\downarrow} \rangle \right]$$

(equivalent sublattices A and B)

↪ specific and sharp signal at/below  $T_N$

but: need right orientation,  $T \lesssim T_N$

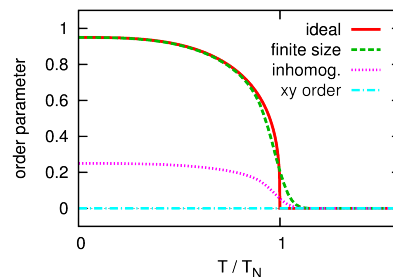
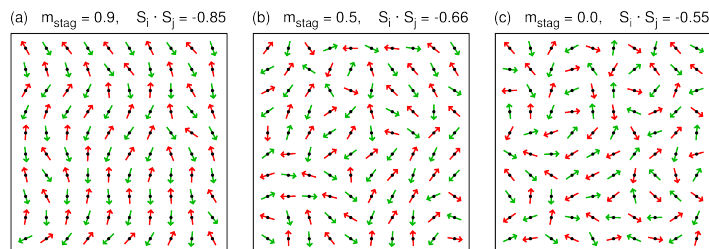


Illustration:

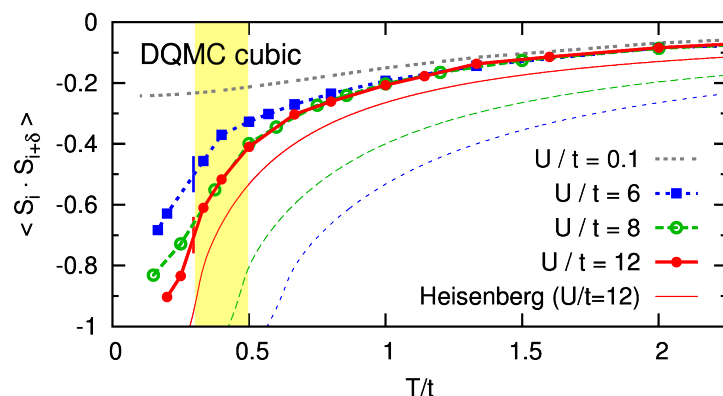
weak order not apparent in small snapshot

↪ look at NN correlations?



## Current experimental focus: nearest-neighbor spin correlation function

Modulation spectroscopy (Esslinger group), super-lattice (Bloch group)



Note: strong (universal) high-temperature tails, monotonous

No distinct features at Néel temperature ( $\approx 0.3t$ )

Interesting spin physics above  $T_N$ , not visible in NN correlations

# Néel transition of trapped fermions on cubic optical lattice at (real-space) DMFT level

[Gorelik et al., PRL **105**, 065301 (2010)]



Elena Gorelik  
Univ. Mainz



Walter Hofstetter  
Univ. Frankfurt

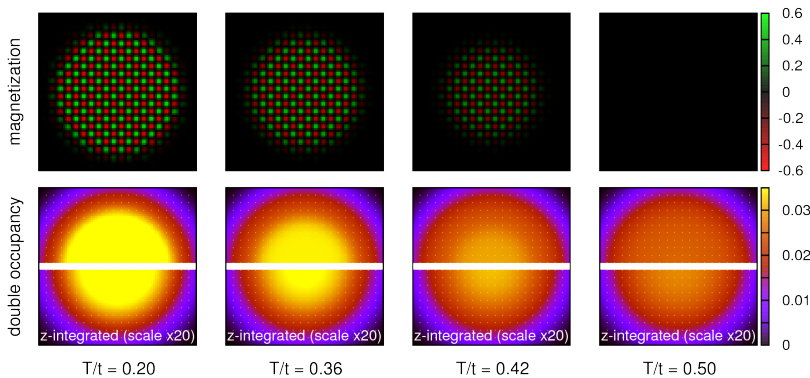


Irakli Titvinidze  
Univ. Hamburg



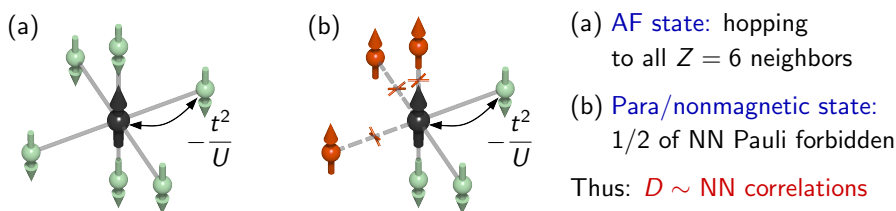
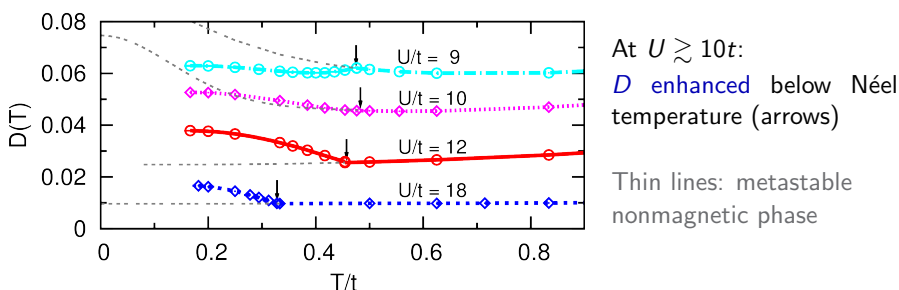
Michiel Snoek  
Univ. Amsterdam

## RDMFT-QMC results for cubic lattice ( $V = 0.05t$ , $U = W = 12t$ )



Proposal: **enhanced double occupancy** (i.e. interaction energy)  
as a signature of antiferromagnetic order/correlations **at strong coupling**  
[Gorelik, Titvinidze, Hofstetter, Snoek, Blümer, PRL (2010)]

## DMFT-QMC estimates of double occupancy $D$ at half filling



Exact GS relation (all  $d$ ) [Takahashi, 1977]:  $D_0 = \frac{Zt^2}{2U^2} (1 - \langle \sigma_i \cdot \sigma_j \rangle) + \mathcal{O}\left(\frac{t^4}{U^4}\right)$

# Effects of non-local correlations? DMFT versus direct QMC + BA

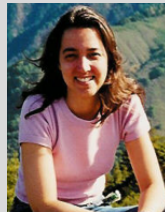
[Gorelik, Rost, Paiva, Scalettar, Klümper, NB, arXiv:1105.3356]



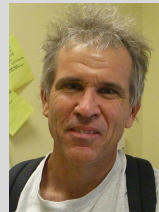
Elena Gorelik  
Univ. Mainz



Andreas Klümper  
Univ. Wuppertal

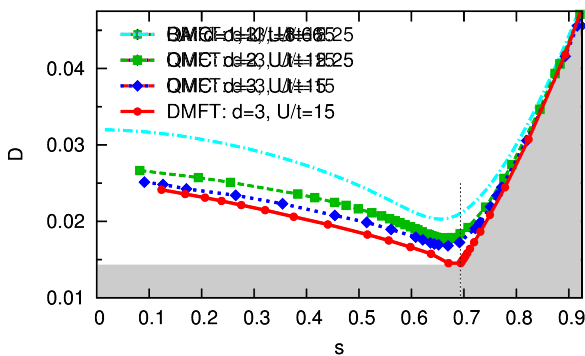


Thereza Paiva  
Rio de Janeiro



Richard Scalettar  
UC Davis

## Double occupancy as a universal measure of AF correlations + entropy



DMFT PT at  $s \approx \log(2)$

No features seen at  $d = 3$  Néel transition

Minimum of  $D(s)$  at  $s \approx \log 2$  for all  $d$ !

Interesting physics at  $\log(2)/2 \lesssim s \lesssim \log(2)$  !?

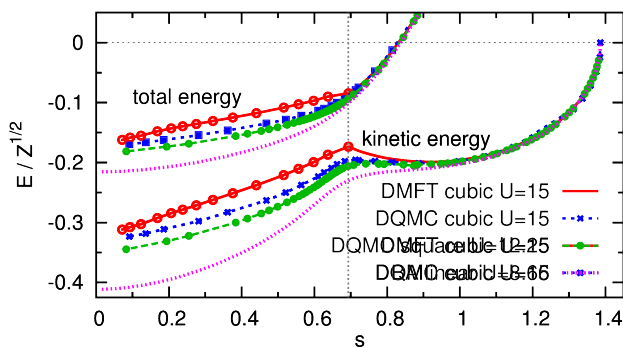
AF enhancement of  $D$  is larger

in lower dimensions:

$$D_0 = (1 - \langle \sigma_i \cdot \sigma_j \rangle) Z \frac{t^2}{2U^2} + \mathcal{O}(t^4/U^4)$$

$$\langle \sigma_i \cdot \sigma_j \rangle_0 = \begin{cases} -1.00 & \text{DMFT} \\ -1.20 & (d = 3) \\ -1.34 & (d = 2) \\ -1.77 & (d = 1) \end{cases}$$

## Related observable: kinetic energy



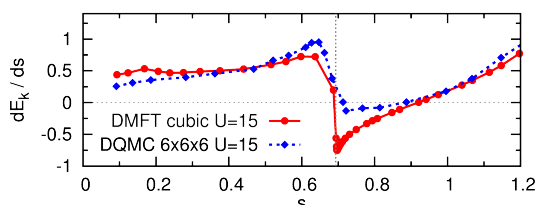
Kink in kinetic energy (only) at  $s \approx \log(2)$

Negative slope at  $s \gtrsim \log(2)$  in  $d = 3$ ?

Yes!

Check:  $dE_{\text{tot}}/ds \geq 0$

$d = 2$  similar to  $d = 3$

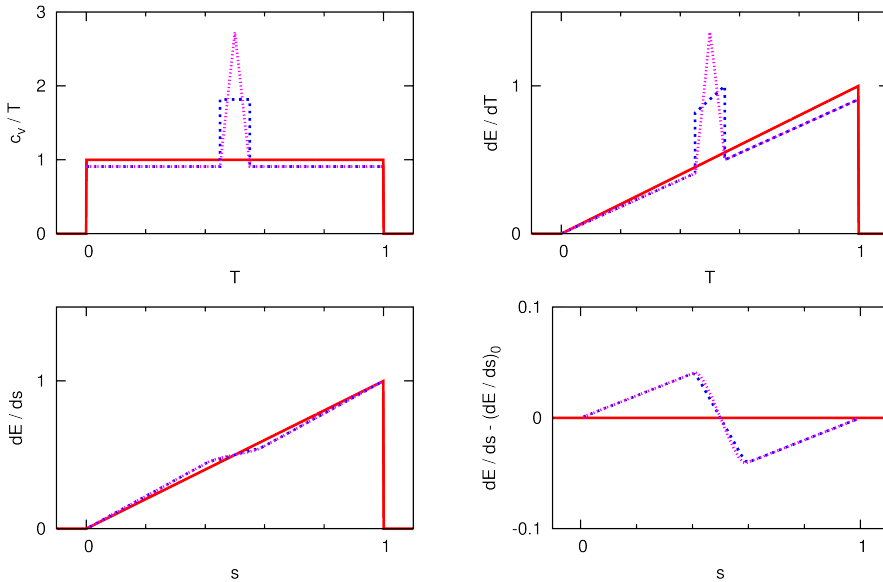


Why no signature at  $d = 3$  phase transition (Ehrenfest)?

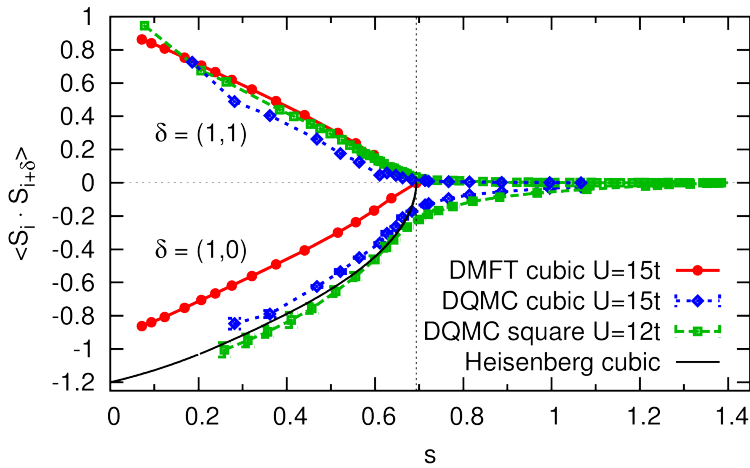
Finite-size effects, num. noise

Specific heat smoothed vs.  $s$

## Illustration: from $dE/dT$ to $dE/ds$

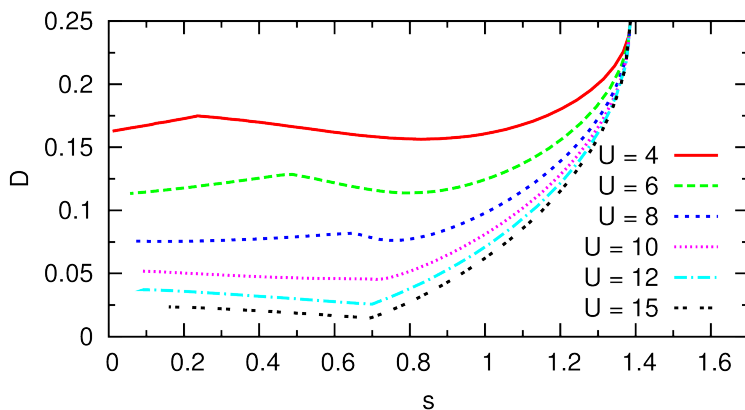


## Spin correlation functions: what range is needed?



NNN spin correlation function signals: Heisenberg regime, low entropy

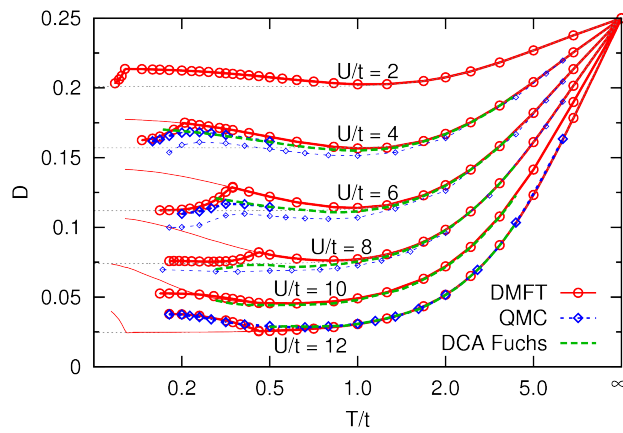
## Weak and moderate coupling: qualitatively new physics



AF destroys Fermi liquid low- $T$  enhancement of  $D$  at  $U \lesssim 10$

DMFT phase transition at  $s = \log(2)$  only in strong coupling limit!

## Comparison DMFT – direct QMC for the 3d cubic lattice ( $n = 1$ )

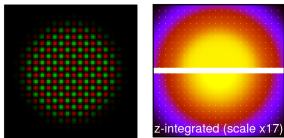


Excellent general agreement DMFT  $\leftrightarrow$  QMC, even at small  $U$

DCA study [Fuchs et al., PRL (2011)] misses AF signatures

Typical QMC discretization errors (thin lines) larger than DMFT deviations!

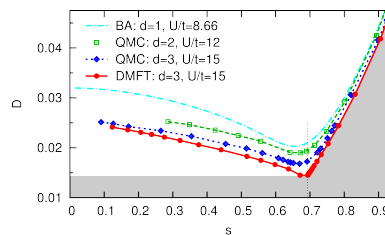
## Summary and Outlook



RDMFT: accurate approach for inhomogeneous correlated Fermi systems (cold atoms or materials)

Double occupancy: universal probe of AF correlations and entropy

Relevant entropy scale for ultracold experiments (local probes):  $s \approx \log(2)$



Frustration: cold atoms [Esslinger group!] and materials

Inhomogeneities, e.g. impurity atom in harmonic trap

Inequivalent flavors ( $\sim$  orbital-selective Mott transitions), multi-flavor